Time and Frequency Synchronisation in Optical Wireless OFDM Networks

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***Abstract*—This paper analyses the impact of imperfect synchronisation in optical orthogonal frequency division multiplexing (OFDM) systems utilising intensity modulation (IM) / direct detection (DD). The use of IM/DD technique inherently eliminates the problem of carrier frequency offset. Therefore, time-frequency synchronisation in optical OFDM system reduces to frame synchronisation and sampling clock synchronisation. Sampling clock offset causes irreducible intercarrier interference (ICI) and symbol timing offset. A technique to mitigate the impact of sampling clock offset is proposed in this paper. In the proposed approach, the received signal is oversampled at twice the Nyquist rate and the odd samples are punctured. The symbol timing offset is estimated using pilots and when the symbol timing offset exceeds ± 0.5 of sample duration, the puncturing pattern flips between puncturing the odd series and the even series. This in effect, constraints the symbol timing offset within ± 1 sample duration. The residual symbol offset can be perfectly corrected using linear phase equaliser. The results show that the proposed method attains a performance comparable to the system that is ideally synchronised provided that the sampling clock offset is lower than 50 ppm.**

I. Introduction

Transmitting data using optical wireless has been identified as a promising technique for short range communications in areas containing critical systems, such as aircraft cabins and hospitals, where radio frequency (RF)-based transmissions are traditionally prohibited to avoid interference to critical systems [1–3]. The state-of-the-art [4] for data transmission using visible light utilises an on-off keying (OOK), which is rather inflexible when it comes to serving multiple users with variable data rate requirements. Moreover, in a system using OOK it is impossible to exploit full channel capacity because the data rate does not scale with the signal-to-interference-plus-noise ratio (SINR). This problem is addressed using OFDM for modulating the intensity of the light emitting diode (LED) which can be detected using a photodiode (PD). OFDM-based optical wireless systems [5, 6] enable flexible allocation of bandwidth among competing users and allow adaptive selection of modulation and coding schemes for achieving the data rates that correspond to traffic demands and prevalent channel conditions at the receiver.

Data transmission using OFDM relies on the fact that the subcarriers are orthogonal to one another at the sampling instant. The orthogonality among the subcarriers in an OFDM system may be lost due to carrier frequency offset and/or the sampling clock offset. For OFDM systems using heterodyne receivers, the carrier frequency offset [7, 8] arises either from frequency mismatch between the local oscillators at the transmitter and the receiver or due to Doppler shift (in mobile environment). Carrier frequency offset is inherently absent in optical wireless systems utilising IM for transmission and DD for reception. Hence, the focus of this paper is on synchronising the sampling clock. The performance of an optical OFDM system may deteriorate due to two imperfections. First, the start of the OFDM symbol at the receiver may either lead or lag behind the start of the OFDM symbol at the transmitter, which is termed as 'symbol timing offset' and the clock frequency of the transmitter and the receiver may deviate from the quoted sampling frequency, which is termed as 'sampling frequency offset'.

Two distinct approaches have been proposed in the literature for mitigating the impacts of the sampling frequency offset. The first approach operates in continuous time and adjusts the sampling clock oscillator directly using a closed-loop feedback [9, 10]. The second approach operates in discrete time and considers mitigating the impact of sampling clock offset rather than adjusting the sampling clock itself. To this end, the time offset is estimated using pilot symbols and the phase of the received samples is corrected using a linear equaliser. Provided that the transmitter and the receiver clocks are not perfectly synchronised, this causes symbol timing offset, which is illustrated using Fig. 1. The symbol timing offset increases as time progresses. Ultimately, there will come a time instant when either a sample will be missed or an extra sample will be taken within an OFDM symbol duration. In such scenario, a time sample is duplicated or discarded, respectively, depending on whether a sample was gained or missed [9–13]. It should be noted that mitigating the impacts of clock offset using the analog approach requires reconfigurable clocks that can be precisely tuned, which increase the cost of the receivers. Expensive tunable oscillators can be avoided if the phase correction can be implemented in the discrete frequency domain. Therefore, a discrete time approach is considered for sampling clock synchronisation in this paper.

The underlying assumption behind the proposed method is that the ICI arising from sampling clock offset can usually be neglected provided that the sampling clock frequency offset is sufficiently small (typically in the order of 100 ppm or lower). The symbol timing offset causes a linear phase shift in the received constellation (as seen in Fig. 2), which can be

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$$Y_{i,l} = X_{i,l}H_{i,l}\exp\left(\frac{j\pi(2i(lN_{\text{os}}-N)+(i\Delta))}{N}\right)\left(\frac{\sin\left(\pi(i\Delta)\right)}{\sin\left(\frac{\pi}{N}(i\Delta)\right)}\right) + \sum_{\substack{k=-N/2\\k\neq i}}^{N/2-1} X_{k,l}H_{k,l}\exp\left(\frac{j\pi(2k(lN_{\text{os}}-N)+(k(\Delta+1)-i))}{N}\right)\left(\frac{\sin\left(\pi(k\Delta+k-i)\right)}{\sin\left(\frac{\pi}{N}(k\Delta+k-i)\right)}\right) \quad (4)$$

detected using pilots and corrected using a linear equaliser, until a sample is missed or gained. To avoid missing a sample or gaining an extra sample, it is proposed that the received signal should be oversampled at twice the Nyquist rate and initially the odd samples are punctured and the even samples are processed. The time offset is estimated using pilot symbols distributed uniformly within an OFDM symbol. Each time the absolute value of the estimated symbol timing offset becomes larger than half of the time interval between two consecutive samples, the receiver switches from puncturing odd samples to puncturing even samples and vice versa. This in effect, avoids missing or gaining extra samples and the linear equaliser corrects the phase rotation due to symbol timing offset.

The remainder of the paper is arranged as follows. The demodulated OFDM signal in presence of sampling clock offset is mathematically modelled in Section II. The proposed approach for addressing problems arising from sampling frequency offset is discussed in Section III. The simulation results are presented in Section IV and the conclusions are drawn in Section V.

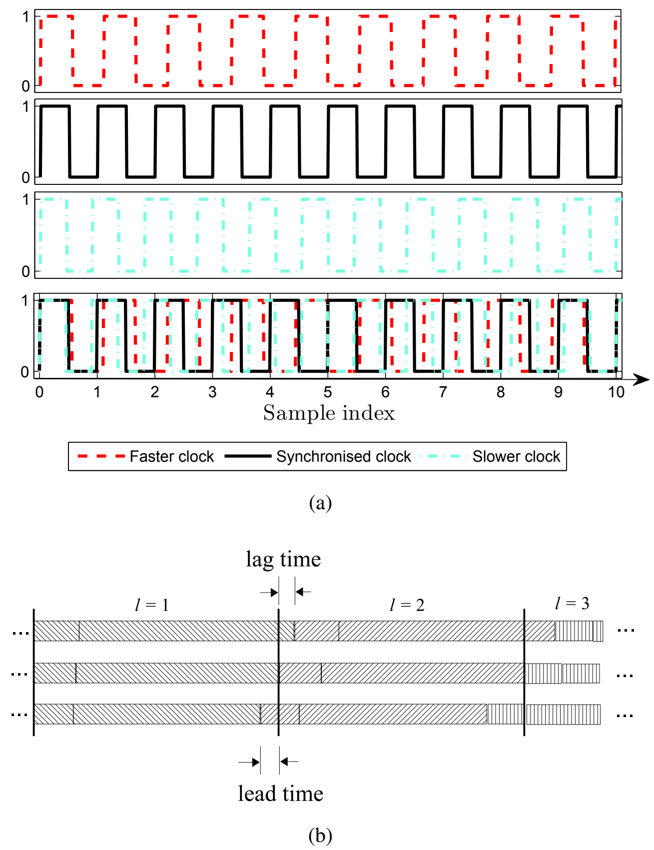

Fig. 1. Illustration of sampling clock frequency mismatch (a). Due to mismatch between the transmitter clock and receiver clock, the start of OFDM symbol at the transmitter leads or lags behind the start of OFDM symbol at the receiver (b). The clock frequency mismatch is exaggerated in this illustration for clarity. In practice, offsets are usually lower than 100 ppm.

II. System Model

Let $X_{k,l}$ denote the baseband symbol transmitted on the k^{th} subcarrier during the l^{th} symbol. Assuming $t=0$ denotes the start of the first OFDM symbol ($l=1$), the l^{th} OFDM symbol begins at $t-(lN_{\text{os}}-N)T_{\text{tx}}$, where $N_{\text{os}}=N+N_{\text{cp}}$ is the total number of OFDM time samples taken per OFDM symbol, and the $T_{\text{tx}}=1/B$ is the period of the transmitter clock and B is the signal bandwidth. Likewise N is the number of subcarriers in the OFDM symbol and N_{cp} is the length of the cyclic prefix (*i.e.* the number of samples within the cyclic prefix when sampled at the Nyquist rate), which is larger than the channel impulse response. With the above notations, the transmitted signal at time instant t is expressed as

$$x(t) = \frac{1}{N}\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X_{k,l}\exp\left(\frac{j2\pi k\left(t-(lN_{\text{os}}-N)T_{\text{tx}}\right)}{NT_{\text{tx}}}\right). \quad (1)$$

In a practical communication link, the transmitter and the receiver clock may not be identical. Provided that $T_{\text{tx}}\neq T_{\text{rx}}$, where T_{rx} is sampling clock period of the receiver, the n^{th} sample of the l^{th} OFDM at the receiver would be taken at $t_{n,l}$ given by $t_{n,l}=(lN_{\text{os}}-N)T_{\text{rx}}$. This would cause either a lead or a lag in OFDM symbol timing, as depicted in Fig. 1. Such symbol timing offset introduces linear phase in the received signal, and non-linear ICI, which shall be demonstrated shortly. To this end, the sampled signal is expressed as

$$\begin{aligned} y_l[n] &= \mathbf{h}_l \otimes \frac{1}{N}\sum_{k=-N/2}^{N/2-1} X_{k,l} \exp\left(\frac{j2\pi k\left(t_{n,l}-(lN_{\text{os}}-N)T_{\text{tx}}\right)}{NT_{\text{tx}}}\right)+\Omega_n \quad (2)\\ &= \mathbf{h}_l \otimes \frac{1}{N}\sum_{k=-N/2}^{N/2-1} X_{k,l}\exp\left(\frac{j2\pi kn(1+\Delta)}{N}\right) \exp\left(\frac{j2\pi k(lN_{\text{os}}-N)\Delta}{N}\right)+\Omega_n, \quad (3)\end{aligned}$$

where $\mathbf{h}_l$ is the channel impulse response, $\otimes$ is the convolution operator, Δ represents the offset between the transmitter and the receiver clock with respect to the transmitter expressed

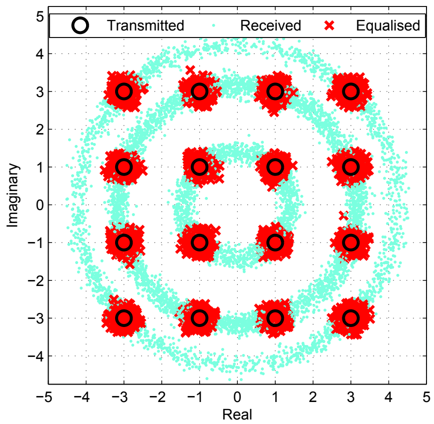

Fig. 2. Transmitted, received and equalised signal constellation assuming 1024 subcarriers, OFDM symbol timing offset equal to half of the sample duration, a sampling clock offset of 50 ppm and an SNR of 25 dB.

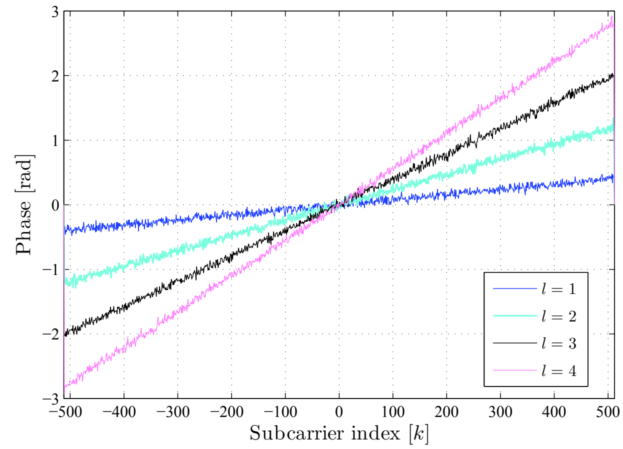

Fig. 3. Depiction of phase rotation on the received signal as a function of subcarrier index. Phase equalisation is linear and unambiguous as long as the symbol timing offset is less than ± 1 sample duration.

as $\Delta = \frac{T_{\text{rx}} - T_{\text{tx}}}{T_{\text{tx}}}$ and Ω_n is the additive white Gaussian noise (AWGN) component in the received signal. The received signal after performing discrete Fourier transform (DFT) operation is given by (4).

The phase rotation of the intended signal on the i^{th} subcarrier of l^{th} OFDM symbol is given by

$$\varphi_{i,l} = \frac{j\pi(2i(lN_{\text{os}} - N) + (i\Delta))}{N}, \tag{5}$$

which can be estimated by transmitting a pilot symbol on the i^{th} subcarrier. Note that for small Δ, the contribution of $i\Delta$ to (5) can be neglected. Therefore, the phase rotation of the received signal is dominated by the OFDM symbol timing offset, which can be corrected using a linear equaliser.

III. Algorithm and Benchmark

The algorithm proposed in this paper for addressing the sampling clock offset ensures that no time samples are missed or gained. In addition, it also uses a pilot aided linear equalisation for correcting the phase induced due to symbol timing offset. The details of the algorithm are discussed in the following.

A. Estimation and correction of symbol timing

The time offset is estimated using pilots distributed evenly across the frequency domain. Since the time offset needs to be continuously monitored, the pilots are repeated in each OFDM symbol. Assuming that $X_{k,l}$ is the baseband symbol (pilot) transmitted on the k^{th} subcarrier during the l^{th} OFDM symbol, the channel transfer function for the k^{th} subcarrier is estimated as

$$H_{k,l} = Y_{k,l}/X_{k,l}\,. \tag{6}$$

The gradient of the phase (shown in Fig. 3) between any two subcarriers can be estimated using linear approximation, given by

$$\Delta_\varphi = \frac{\angle(H_{k,l}) - \angle(H_{k',l})}{k - k'}, \tag{7}$$

where $k \neq k'$. Note that the $Y_{k,l}$ differs from the scaled version of $X_{k,l}$ due to ICI and AWGN noise. Clearly, the estimation given by (7) may not be precise if only closely located subcarriers are used to estimate the gradient of the time offset. Therefore, the pilots for estimating the gradient are scattered uniformly within the OFDM symbol and a least square error approach is used for estimating the time offset to improve the accuracy of the estimate. The estimated time offset is used to adjust the symbol timing, which will be discussed in Section III-B. Assuming that $|H_{k,l}| \approx |H_{k',l}|$ for all subcarriers, which is a reasonable assumption for optical OFDM as long as the sampling clock offset is not very large, the equalised baseband symbol is given by

$$\hat{X}_{k,l} = \frac{Y_{k,l}}{|h_{k,l}|} e^{-j\Delta_\varphi k}\,. \tag{8}$$

The equalised symbol is further processed at the receiver to retrieve the transmitted data stream.

B. Symbol timing adjustment

As discussed earlier, the mismatch between the clock frequencies at the digital-to-analog (D/A) and analog-to-digital (A/D) converter increases the symbol timing offset progressively in each OFDM symbol. Ultimately, the A/D either misses a time sample or obtains an extra time sample within the duration of an OFDM symbol with respect to the transmitter clock. Hence, the symbol timing needs to be periodically adjusted to ensure that the transmitter and the receiver are synchronised. To this end, the time offset is estimated by solving

$$\hat{T}_{\text{os}} = \frac{\log(\varphi_{k,l})}{j\left(\frac{2\pi}{N}\right) l}\,. \tag{9}$$

Note that (9) is not a unique solution because the phase is periodic with N/k. Consequently, the placement of subcarrier determines whether the symbol timing offset given by (9) is unambiguous. In particular, the pilots located at subcarrier indices $N/2$ or $-N/2$ can resolve symbol time offset of up to one sample duration. Likewise, the pilots carried by subcarrier with indices $N/4$ or $-N/4$ can resolve an offset of up to two sample durations without ambiguity. Therefore, the pilots have to be placed such that the maximum possible symbol timing offset envisioned in the system are resolved without ambiguity or the the symbol timing offset T_{os} must be constrained in the range $[-1/B, 1/B]$ so as to avoid ambiguity in time offset estimation if the pilots are distributed throughout the system bandwidth.

In the proposed approach, we assume that coarse synchronisation is carried out using threshold comparison using a known pseudo noise sequence, in particular a Barker code [14]. Therefore, we assume that the symbol timing offset is constrained in the range $[-1/B, 1/B]$. To constrain the symbol timing offset in the aforesaid interval, the received signal is oversampled at twice the Nyquist rate. The oversampled signal is denoted $z[n]$. The samples that are passed on for baseband processing is given by

$$y[n] = z[2n + \delta]\,, \tag{10}$$

where δ is an offset that determines whether odd or even samples obtained from the oversampled signal will be used for processing. The δ parameter is adjusted as follows

$$\delta = \begin{cases} \delta + 1\,, & \hat{T}_{\mathrm{os}} \geq 0.5\,T_{\mathrm{tx}} \\ \delta - 1\,, & \hat{T}_{\mathrm{os}} \leq -0.5\,T_{\mathrm{tx}} \\ \delta\,, & \text{otherwise.} \end{cases} \tag{11}$$

C. Benchmark System

An OFDM system performing phase equalisation aided by pilots is used as a benchmark. The arrangement of pilot symbols and phase equalisation algorithm in the benchmark system is identical to the proposed approach. The only difference is that in the benchmark system the symbol timing offset is not corrected. This allows to assess the improvements achieved by performing of sampling offset correction in OFDM system, provided that the sampling clock synchronisation is imperfect.

IV. Results and Discussions

The performance of the proposed approach is compared against the benchmark in Fig. 4 in terms of average bit error ratio (BER) as a function of time. Both the transmitter and the receiver are assumed to be perfectly time synchronised at $t = 0$ but the time offset increases as time progresses due to sampling frequency offset error. For the results presented in Fig. 4, OFDM transmission using 1024 subcarriers is considered. The modulation format of 16-quadrature amplitude modulation (QAM) (uncoded) and an SNR of 20 dB at the input of the A/D converter are assumed. The results show that the proposed approach attains roughly the same performance as the ideal scenario where the clocks at the transmitter and the receiver are synchronised, provided that the sampling clock offset is lower than 50 ppm. For the considered parameters, both the proposed approach and the ideal case achieve a BER of roughly 10^{-5} assuming an SNR of 20 dB at the input of A/D converter. In such scenario, the BER floor is determined by the noise level at the analog frontend. However, the BER increases with an increase in the sampling clock frequency offset because the ICI is no longer negligible when the sampling frequency offset increases. From this result, it can be concluded that the clock offsets should not exceed 100 ppm in order to avoid detrimental ICI at the receiver.

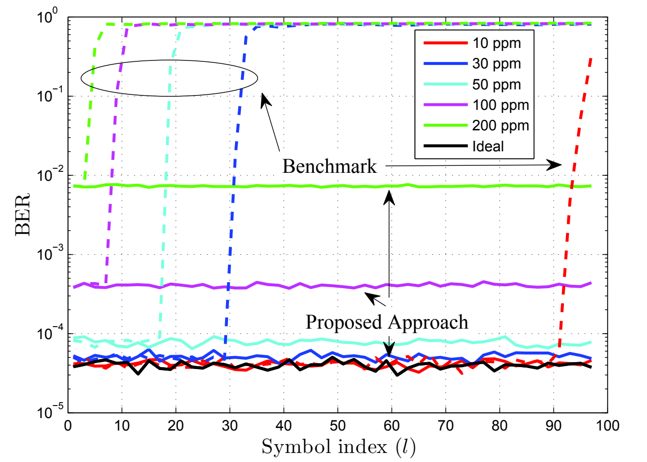

Fig. 4. Comparison of BER as a function of number of OFDM symbol index for different values of sampling clock offset obtained 16-QAM for baseband modulation and utilising 1024 subcarriers

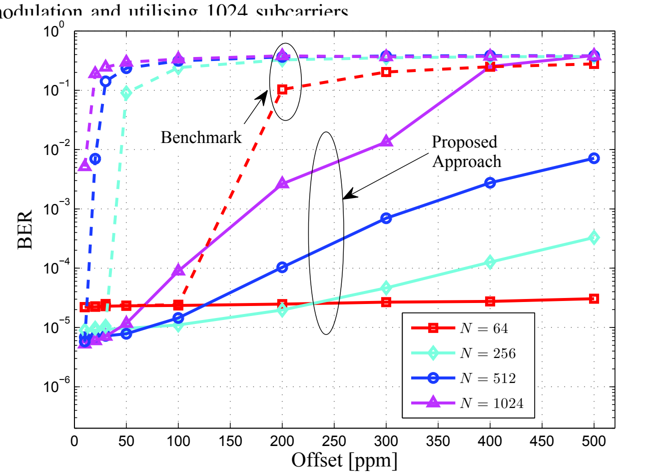

Fig. 5. Comparison of average of BER taken over 100 OFDM symbols as a function of sampling clock offset for different number of subcarriers.

However, with the benchmark system, the BER jumps roughly to 0.5 once the A/D converter misses or gains one time sample, which happens due to accumulation of relative errors between the transmitter and receiver. Once the sample is missed, the estimation of time offset becomes ambiguous as discussed earlier. Moreover, provided that the phase offset induced due to symbol timing offset is almost 2π on the pilot subcarriers, the noise can further push the phase offset towards an ambiguous regime, thereby causing incorrect phase correction which results in bit errors. The proposed approach, by contrast, maintains the symbol timing offset within the region where linear equalisation correct the phase rotation without ambiguity. Therefore, the performance of the proposed approach is identical to that of the synchronised OFDM system (ideal) in absence of detrimental ICI.

Fig. 5 compares the mean BER obtained within 100 OFDM symbols as a function of sampling clock offset values for different fast Fourier transform (FFT) sizes. These results establish a relationship between the number of subcarriers in an OFDM symbol and the maximum sampling clock offset that can be tolerated with the proposed approach whilst maintaining a certain average BER. The results show that the BER increases with an increase in sampling frequency offset, as expected. Furthermore, BER performance degrades with an increase in the number of subcarriers for a fixed clock offset value. In particular, for a sampling clock offset of 100 ppm, the BER increases by an order of magnitude for $N = 1024$ compared to those achieved using $N = 512$. Furthermore, it can also be noted that for $N = 64$, the BER performance is roughly constant until sampling clock offset of 500 ppm. This demonstrates that cheap (and therefore less precise) oscillators can still be used in A/D converters whilst still maintaining the performance comparable to the ideal case.

The average BER achieved with the proposed approach is compared against that of the benchmark system and an ideally synchronised system in Fig. 6 for a system using uncoded 16-QAM and 1024 subcarriers. The result show that the BER performance degrades with an increase in sampling clock offset due to ICI, as expected. The results show that the proposed approach achieves the performance comparable to that of an ideal system as long as the the clock frequency offset is smaller than 50 ppm for the considered set of parameters. By contrast, the average BER is in the order of 10^{-2} for the benchmark even when the offset is merely 10 ppm. The error arises due to uncertainty in estimating the symbol timing offset when the symbol timing offset is larger than 1 sample duration, since the symbol timing offsets are not corrected with the benchmark system. Such uncertainties are eliminated by the proposed approach since it keeps the symbol timing offset locked within ± 1 sample duration. Thus, the phase rotation is always resolved without any ambiguity and the performance of the proposed approach is roughly the same as that of an ideal system as long as ICI is not dominant.

V. Conclusions

In this paper, an algorithm for correcting the sampling clock offset in an optical wireless OFDM system is proposed. The results show that the proposed algorithm significantly outperforms the benchmark system where linear phase equalisation is carried out. It is demonstrated that the proposed method attains the performance of a perfectly synchronised system (ideal case), given that the offset is limited to 50 ppm. The proposed algorithm requires oversampling at twice the Nyquist rate and puncturing half of the samples, in addition to the steps performed in the benchmark system. Therefore, the proposed algorithm can be easily implemented in digital hardware. The restriction that the proposed approach could impose is the availability of A/D converters that can sample the incoming signal at twice the Nyquist rate, which imposes restrictions on the bandwidth of the signal that can be transmitted. However, the actual bottleneck in an optical wireless system typically arises from the bandwidth of the analog frontends, which limit the bandwidth of the signal that can be transmitted. Hence, the proposed approach can be used for correcting the sampling clock frequency mismatch between the transmitter and the receiver in an optical wireless OFDM system.

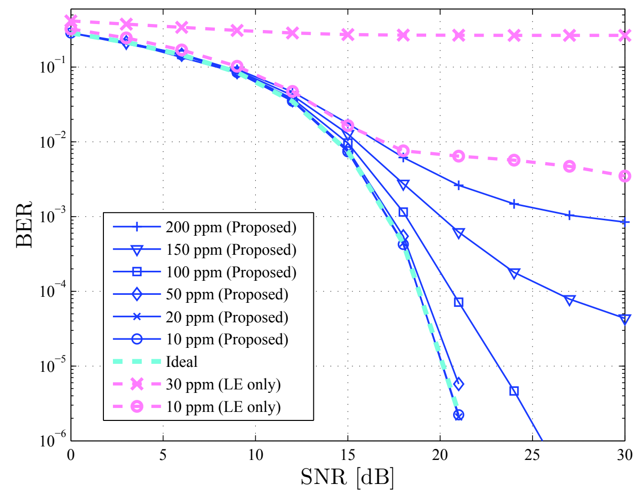

Fig. 6. Comparison of the proposed approach against the benchmark system in terms of BER performance achieved using uncoded 16-QAM with different values of SNR.

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