

Enhanced Subcarrier Index Modulation (SIM) OFDM

Dobroslav Tsonev, Sinan Sinanovic and Harald Haas

Institute for Digital Communications, Joint Research Institute for Signal and Image Processing,

The University of Edinburgh, EH9 3JL, Edinburgh, UK

Email: {dtonev,s.sinanovic,h.haas}@ed.ac.uk

Abstract—A novel modulation technique coined SIM-OFDM was recently proposed. SIM-OFDM uses different frequency carrier states to convey information and leads to increased performance in comparison to conventional OFDM. Additionally, its innovative structure can lead to a decrease of the peak system power, which is highly beneficial in the context of optical wireless communication. One of the issues of the original SIM-OFDM scheme is a potential bit error propagation which could lead to significant burst errors. The current paper proposes a modified technique which avoids bit error propagation whilst retaining the benefits of the concept.

I. INTRODUCTION

SIM-OFDM is a modification of the classical OFDM modulation scheme. In OFDM a number of different frequency carriers are modulated with a signal from a scheme such as Quadrature Amplitude Modulation (QAM). The novel approach of SIM-OFDM tries to exploit an additional “new” dimension in the OFDM frame coming from the state of each subcarrier – active or inactive. This additional dimension is employed to transmit information in an On-Off Keying (OOK) fashion. A detailed description of SIM-OFDM can be found in [1].

The motivation behind this new concept lies in an attempt to optimize power usage, which is crucial in the current climate of “green communication systems” [2]. Each active carrier receives the energy of an M -QAM symbol and the energy of the additional bit encoded in OOK fashion. The individual performances of QAM and OOK are thus improved. Overall, this leads to performance improvement of SIM-OFDM on an energy-per-bit basis.

SIM-OFDM is still a rather unexplored topic. An analytical approach towards deriving its performance in a fading channel is given in [1]. The same paper reports that SIM-OFDM has the potential to outperform the traditional OFDM modulation scheme in terms of bit error rate (BER). However, to the best of our knowledge, a complete analytical description of its BER performance does not exist in literature. SIM-OFDM suffers from bit error propagation in the presence of Additive White Gaussian Noise (AWGN). This work provides a proposition for a modified scheme which solves the issue of bit error propagation. A further benefit of our modified approach is reduction of the Peak-to-average power ratio (PAPR) which comes from the new enhanced structure of SIM-OFDM.

The rest of this paper is organized as follows. Section II gives a brief description of the SIM-OFDM modulation scheme and its demodulator. Section III describes the new enhanced modulation scheme. Section IV shows an analytical BER derivation of the enhanced SIM-OFDM. Section V

presents the numerical simulation results, which demonstrate the close match between analysis and Monte Carlo simulations of the system performance. Section VI discusses the PAPR of the modified approach. Finally, section VII gives concluding remarks to the topic.

II. SIM-OFDM

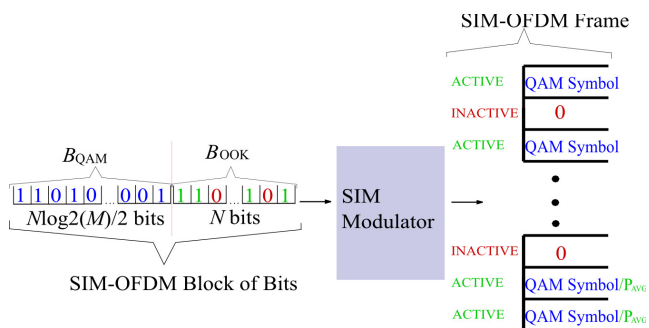


Fig. 1. SIM-OFDM carrier modulation scheme. Assumption for this example is that 1 is the majority bit in B_{OOK}

In SIM-OFDM the incoming bit stream is divided into blocks of bits, each having a length of $N(\frac{\log_2(M)}{2} + 1)$, where N is the number of subcarriers, and M is the constellation size of the respective M -QAM modulation scheme. Each of these blocks is divided into two parts. The first N bits of the block form a subblock, which in the rest of this paper will be referred to as B_{OOK} . The remaining $N \frac{\log_2(M)}{2}$ bits form a second subblock, which will be referred to as B_{QAM} . This is further illustrated in Fig. 1. Before transmission B_{OOK} is inspected and the majority bit type is determined by checking which bit value, 1 or 0, has more occurrences. This is done because all carriers which have the same position inside the OFDM frame as the bits from the majority bit type in B_{OOK} will be classified as “active”, and the rest will be classified as “inactive”. Inactive carriers are given the amplitude value $0 + j0$ where $j = \sqrt{-1}$. The first $\frac{N}{2}$ active carriers will be given amplitude values corresponding to M -QAM constellation symbols necessary to encode B_{QAM} . The remaining active carriers can be used to signal to the receiver what the majority bit type of B_{OOK} is, and they will be assigned a signal whose power is equal to the average power for the given M -QAM scheme. This is followed by an N -point Inverse Fast Fourier Transform (IFFT) in order to obtain the time-domain signal. A slight modification of SIM-OFDM suggests that majority bit type can be signaled either through secure communication

channels, or by reserving one particular frequency carrier and transmitting the desired value with sufficiently high signal-to-noise ratio (SNR). This leads to a slight improvement in the SIM-OFDM performance.

Upon reception, the signal is transformed using a Fast Fourier Transform as in traditional OFDM. Then all subcarriers are inspected. Those subcarriers whose power is above a certain threshold are marked as active, and the rest are marked as inactive. Then B_{OOK} is reconstructed from the detected states of the carriers and the known majority bit type. Afterwards, the first $\frac{N}{2}$ active carriers are demodulated according to the respective M -QAM scheme in order to reconstruct B_{QAM} .

There are two main issues which limit the performance of SIM-OFDM. First, an OOK detector at the destination requires the usage of a threshold. The threshold, however, needs to be below the power of the M -QAM symbol with the lowest power. Otherwise, any symbol below the threshold would be unrecognizable even in high SNR conditions, and that would lead to an error floor above zero. On the other hand, a low threshold requirement diminishes the advantage of OOK when a higher order M -QAM scheme is used. This happens because even though on average a lot of power is used in a subcarrier for higher order M -QAM, the threshold needs to be below the smallest symbol power. Hence, the system's ability to correctly distinguish subcarrier states does not improve significantly since the threshold cannot change much. Second, an incorrect detection of a carrier state not only leads to incorrect demodulation of the M -QAM symbol it encodes, but also to incorrect demodulation of all subsequent QAM symbols. This happens because an incorrect detection of a carrier state changes the order in B_{QAM} and misplaces all subsequent bits in that part of the frame. This effect has the biggest impact on the BER performance. Effectively, in order to demodulate an M -QAM symbol correctly, it is necessary not only to detect its carrier as active, but also to have detected any previous carriers – active or inactive – correctly. As the total number of carriers, N , increases, the issue becomes worse. The main contribution of this paper is a solution to these two problems.

III. ENHANCED SIM-OFDM

As mentioned in section II, the error propagation becomes worse as the number of subcarriers, N , grows. However, a slight modification in the way B_{OOK} is encoded can limit this effect. Instead of each bit from B_{OOK} being encoded in a single subcarrier state, it can be encoded in the states of two consecutive carriers – a carrier pair. Fig. 2 illustrates the principle of the new encoding scheme. Whenever a 1 is encountered in B_{OOK} , the first carrier of the pair is set as active and the second one as passive. Whenever a 0 is encountered in B_{OOK} , the first carrier of the pair is set as passive and the second one as active. Effectively, now the size of B_{OOK} , which can be represented by carrier states, is $\frac{N}{2}$ – half the size than in the original SIM-OFDM scheme. The benefit, however, is that for each pair it is certain that exactly one of the carriers is active. This means that bits from B_{QAM} can no longer be misplaced due to wrong detection of previous subcarrier states. The error that can be made is limited within

each pair of carriers. In addition, there is no longer a need to define a bit-majority in B_{OOK} , and the total number of active carriers is always the same – $\frac{N}{2}$. The number of active carriers within each pair is known and is always one, so there is no need to use a threshold for OOK detection. Instead, the carrier with higher power can be recognized as active, which would lead to better performance in the carrier state detection.

The only disadvantage of the modified scheme, compared to the former SIM-OFDM, is the slightly reduced spectral efficiency. Spectral efficiency of SIM-OFDM, measured in bits/carrier, is

$$\eta_{SIM-OFDM}^{OLD} = \frac{\log_2(M)}{2} + 1, \quad (1)$$

while the spectral efficiency of the modified SIM-OFDM is

$$\eta_{SIM-OFDM}^{NEW} = \frac{\log_2(M)}{2} + \frac{1}{2} \quad (2)$$

For comparison purposes, the spectral efficiency of QAM-encoded OFDM is $\log_2(M)$. This negative characteristic can be mitigated for SIM-OFDM when carrier pairs are expanded to blocks with a total number of subcarriers $L > 2$. For example, if $L = 8$ is used, there are 70 combinations in which 4 out of 8 subcarriers are set as active. This means a total of 6 bits ($2^6 = 64 < 70$) from B_{OOK} can be represented by combinations of active subcarriers in a block of 8 subcarriers, which already yields a better spectral efficiency than in (2). If the number of subcarriers in the block is L , and the number of active carriers within the block is L_a , a general expression for the spectral efficiency becomes

$$\eta_{SIM-OFDM}^{GENERAL} = \frac{L_a \log_2(M)}{L} + \frac{\lceil \log_2(\frac{L!}{L_a!(L-L_a)!}) \rceil}{L} \quad (3)$$

If L_a is chosen to be close to L , the system gets closer to conventional OFDM. If L is chosen to be close to N , and L_a approaches 1, the system starts to resemble PPM. As L approaches N , and L_a approaches $\frac{N}{2}$, the system gets closer to the original SIM-OFDM.

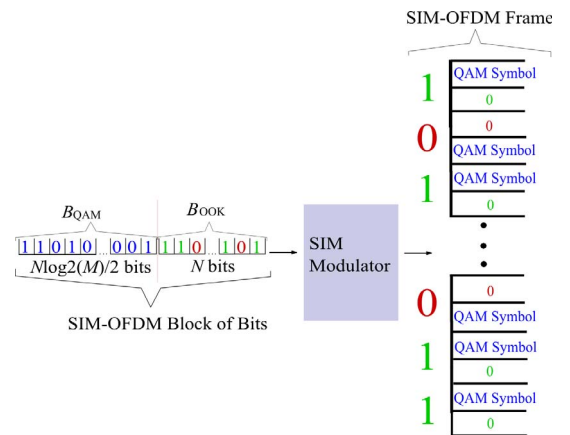


Fig. 2. Modified SIM-OFDM modulation approach.

IV. ENHANCED SIM-OFDM BER DERIVATION

The BER derivation that follows describes the operation of the system for $L_a = 1$ and $L = 2$ in the presence of AWGN.

Analysis for a different combination of L_a and L can be made in an analogous manner. No fading is assumed since it is not an issue in scenarios related to optical wireless communication for which SIM-OFDM is primarily considered. There is no dependence between the separate subcarrier pairs in the enhanced SIM-OFDM frame. Therefore, the BER analysis of one pair applies to the entire system.

Each carrier pair is received with an addition of AWGN. Hence, the distributions of both carrier amplitudes resemble the example illustrated in Fig. 3. The 'X' marks a possible value of the amplitude of the active carrier. In this case, all values, which the inactive carrier may take and still be correctly recognized as inactive, are contained inside the thick circle. Hence, integrating the Gaussian distribution, centered at 0, inside the thick circle would give the probability for correctly distinguishing between the passive and active carrier in the pair. Using some results from [3], this integration is represented by (4), where d is 1 when carriers are correctly distinguished or 0 when they are not. The symbol c stands for the amplitude of the active carrier, and N_o stands for the power of the noise.

$$\begin{aligned}
 P(d = 1|c = x + jy) &= \\
 &= \int_{-\sqrt{|x+jy|^2}}^{\sqrt{|x+jy|^2}} \int_{-\sqrt{|x+jy|^2-b^2}}^{\sqrt{|x+jy|^2-b^2}} \frac{1}{\pi N_o} e^{-\frac{(a)^2+(b)^2}{N_o}} da db \stackrel{[3]}{=} \\
 &\stackrel{[3]}{=} \int_0^{2\pi} \int_0^{\sqrt{|x+jy|^2}} \frac{1}{\pi N_o} e^{-\frac{r^2}{N_o}} r dr d\theta = \\
 &= 1 - e^{-\frac{|x+jy|^2}{N_o}}
 \end{aligned} \tag{4}$$

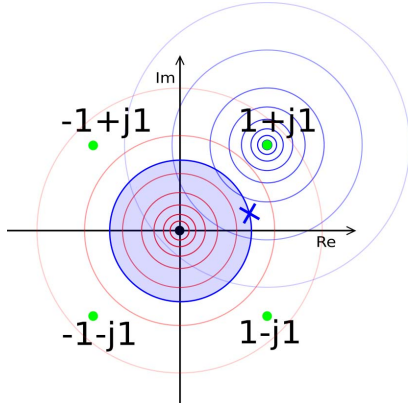


Fig. 3. Contour lines of the distribution of the received carrier pair as a function of the transmitted constellation symbol. Lines are not drawn to scale.

Equation (5) expresses the probability density that the amplitude of the active carrier takes the value $x + jy$. There is an equal probability, $\frac{1}{M}$, that the value of the carrier before the addition of AWGN was equal to any one of the M constellation points. Hence, the probability density that the carrier value at the receiver is equal to $x + jy$ is expressed as a weighted sum of M different probability densities coming from the Gaussian functions centered at the M constellation points. Symbol μ_k marks constellation point number k . Symbols $\Re\{\mu_k\}$ and $\Im\{\mu_k\}$ denote the real and imaginary part of μ_k .

$$\begin{aligned}
 P(c = x + jy) &= \\
 &= \frac{1}{M} \sum_{k=1}^M \frac{1}{\pi N_o} e^{-\frac{(x-\Re\{\mu_k\})^2+(y-\Im\{\mu_k\})^2}{N_o}}
 \end{aligned} \tag{5}$$

Equation (6) expresses the probability for correctly determining the active and inactive subcarrier in a pair. Equation (7) expresses the complementary probability - that the active and inactive subcarrier are incorrectly determined.

$$\begin{aligned}
 P_1 &= P(d = 1) \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(d = 1|c = x + jy)P(c = x + jy)dx dy \\
 &= 1 - \frac{1}{2M} \sum_{k=1}^M e^{-\frac{|\mu_k|^2}{2N_o}}
 \end{aligned} \tag{6}$$

$$P_0 = P(d = 0) = 1 - P(d = 1) \tag{7}$$

Based on the calculated detection probabilities, the BER performance for the system can be calculated. When the carrier states are incorrectly determined, the bit from B_{OOK} , which they encode, is in error. That bit amounts to $\frac{1}{\log_2(M)+1}$ part of all the bits encoded in a carrier pair. Hence, the BER contribution of B_{OOK} is expressed as

$$BER_{B_{OOK}} = P_0 \frac{1}{\log_2(M) + 1} \tag{8}$$

An erroneous detection of a subcarrier state leads to the attempt of M -QAM demodulation of an unmodulated subcarrier, which adds additional errors to the frame. Such erroneous detection occurs with very high probability in the four M -QAM symbols close to 0. Probability that wrong detection occurs for any of the other symbols in the constellation is low in comparison to the probability for those four. Those four symbols encode bits which differ by only one bit on average when the M -QAM constellation is Gray encoded. Hence, in most of the cases only 1 bit will be wrong in B_{QAM} for wrong subcarrier detection. This is especially true for high SNR and high modulation order, M . Hence, the first part of the BER contribution of B_{QAM} is the same as in (8). Whenever a subcarrier state is correctly determined, there still exists the possibility that the QAM demodulation will lead to errors. Subcarrier state detection is not independent from the QAM demodulation process since both depend on the same subcarrier value. However, the Gray mapping of the constellation leads to very similar BER for all symbols, especially for high SNR. This happens because symbols are mostly mistaken for their neighbouring symbols. Hence, the BER of M -QAM, $B_{QAM_{BER}}$ defined in [4], can be used almost independently from the carrier state detection in (9) in order to complete $BER_{B_{QAM}}$ which is the contribution of B_{QAM} to the overall bit error rate. This assumption is confirmed by numerical results in V and should become more accurate as SNR increases since distributions converge towards the original QAM constellation points. The final factor indicates that the bits encoded by M -QAM are $\frac{\log_2(M)}{\log_2(M)+1}$ of

all the bits encoded in a carrier pair.

$$\text{BER}_{B_{QAM}} = P_0 \frac{1}{\log_2(M) + 1} + P_1 M \text{QAM}_{\text{BER}} \frac{\log_2(M)}{\log_2(M) + 1} \quad (9)$$

The final expression for the bit error rate of the modified SIM-OFDM system is as follows:

$$\text{BER}_{\text{SIM-OFDM}} = \text{BER}_{B_{OOK}} + \text{BER}_{B_{QAM}} \quad (10)$$

V. NUMERICAL RESULTS

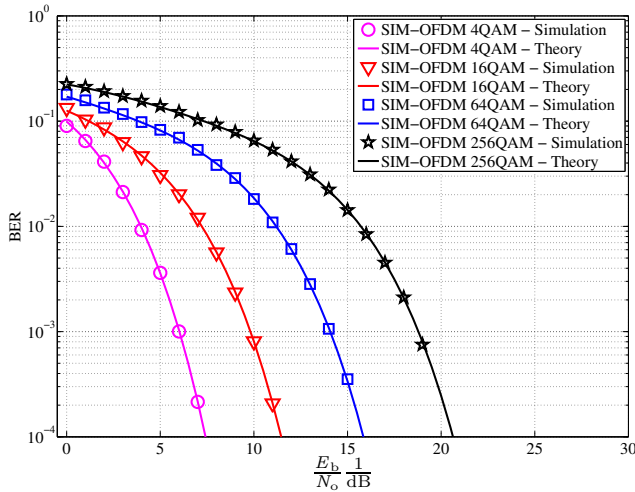


Fig. 4. Comparison between theoretical model and Monte Carlo simulations for modified SIM-OFDM ($L=2$, $L_a=1$) performance in the presence of AWGN

The numerical simulations of the modified SIM-OFDM were performed for $L = 2$ and $L_a = 1$ in accordance with the analysis in section IV. The results that compare the analytical model with Monte Carlo simulations are presented in Fig. 4. The Monte Carlo simulations are obtained for 1000 SIM-OFDM frames with errors.

In order to fairly compare SIM-OFDM with conventional OFDM on a power-per-bit basis, each active carrier should receive the energy of its M -QAM symbol and additional energy to account for the bit encoded in the carrier states. The comparison can be observed in Fig. 5. For the same QAM modulation order, SIM-OFDM requires less energy per bit than OFDM to achieve a target BER, which leads to an energy gain. At the same time, M -QAM SIM-OFDM is less spectrally efficient than M -QAM OFDM as illustrated in (2). This decreases its performance for high orders of M . However, SIM-OFDM effectively improves the BER performance when high spectral efficiency is not required. For example, 4-QAM SIM-OFDM produces a BER curve which is not achievable by 4-QAM OFDM without the use of coding. SIM-OFDM with less active carriers than the presented version – $L = 2$, $L_a = 1$ – has the potential to produce even more energy efficient schemes.

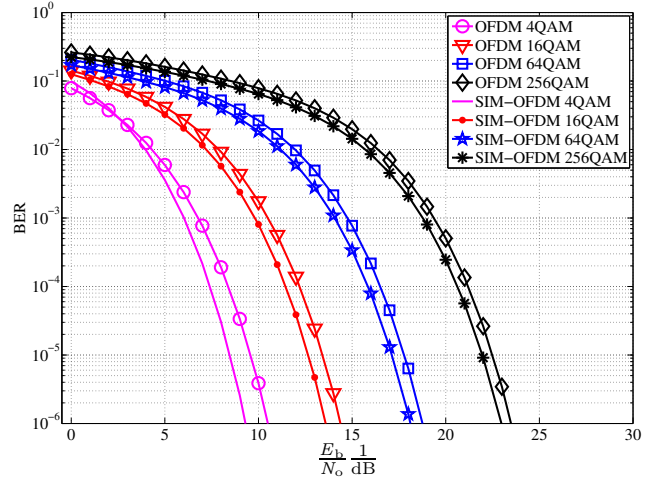


Fig. 5. Comparison between modified SIM-OFDM ($L=2$, $L_a=1$) and conventional OFDM performance in the presence of AWGN

VI. PEAK-TO-AVERAGE POWER RATIO

Conventional OFDM modulation suffers from a high PAPR, which can present a considerable problem to amplifier and quantizer design in the system [4]. This aspect of SIM-OFDM is worth discussing because the structure of the modified SIM-OFDM frame leads to reduction of the PAPR, which is highly beneficial especially in the context of optical wireless systems where power is limited by eye safety regulations [5].

PAPR is defined as the ratio of the maximum achievable power at any point in time, P_{MAX} , and the average power of the signal, P_{AVG} [4]:

$$\text{PAPR} = \frac{P_{\text{MAX}}}{P_{\text{AVG}}} \quad (11)$$

The time domain signal, $x[n]$, to be transmitted in an OFDM system is calculated as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn} \quad (12)$$

where $X[k]$ is the value assigned to the k^{th} subcarrier. In conventional OFDM, the peak power in time occurs at $x[a]$ when

$$X[k] = b e^{-j2\pi ka}, \quad b \in \mathbb{C} \quad (13)$$

The complex exponential in (12) is cancelled by the complex exponential in (13) and the result is a summation of N equal complex numbers b . When two complex numbers with the same power are summed, they yield a result with the highest possible power when both numbers are the same. Fig. 6 illustrates the concept graphically. Hence, (13) guarantees a peak value at $x[a]$ when b has the highest possible power. The maximum power is achieved at $x[0]$ when each subcarrier is modulated with the same constellation symbol with the highest possible power. That symbol is marked as S_{HP} . Then $x[0] = \sqrt{N} S_{\text{HP}}$ and $x[0] x[0]^* = N S_{\text{HP}} S_{\text{HP}}^*$. For a square M -QAM modulation $S_{\text{HP}} = (\sqrt{M} - 1) + j(\sqrt{M} - 1)$ and

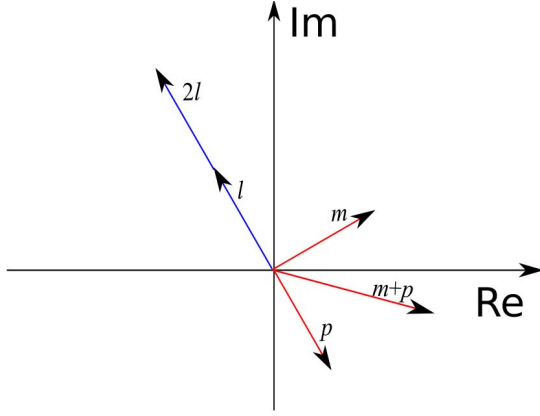


Fig. 6. Graphical representation of sum of complex numbers with the same power - $l, m, p \in \mathbb{C}$; $|l|^2 = |m|^2 = |p|^2$; $|2l|^2 > |m+p|^2$

$S_{\text{HP}} S_{\text{HP}}^* = 2(\sqrt{M} - 1)^2$ [4]. Hence, the maximum power is

$$P_{\text{MAX}} = 2N(\sqrt{M} - 1)^2 \quad (14)$$

The average power is equal to the average frame energy divided by the number of points in time domain. The average frame energy is equal to the average energy per carrier multiplied by the number of carriers. The average energy per carrier is equal to the average energy in an M -QAM symbol. Hence, using [4],

$$P_{\text{AVG}} = \frac{E_{\text{AVG}}}{N} = \frac{E_{\text{AVG}}^{\text{QAM}} N}{N} = E_{\text{AVG}}^{\text{QAM}} \stackrel{[4]}{=} \frac{2(M-1)}{3} \quad (15)$$

From (14) and (15), the PAPR for conventional OFDM can be calculated as

$$\text{PAPR}_{\text{OFDM}} = \frac{P_{\text{MAX}}}{P_{\text{AVG}}} = \frac{3N(\sqrt{M} - 1)^2}{M - 1} = \frac{3N(\sqrt{M} - 1)}{\sqrt{M} + 1} \quad (16)$$

An alternative derivation of (16) can be found in [4].

In SIM-OFDM, a number of subcarriers are “switched off”, *i.e.*, set to 0. A complex exponential, however, never takes the value 0; its power is always 1. Hence, it is not possible to allocate all the signal energy at a single point in time anymore. This already suggests a reduction of the peak power value. Adding zeros to the expression in (12) only means that a smaller number of complex numbers need to be summed in order to obtain $x[n]$. They still need to have the highest possible power and be the same due to the property of complex numbers, described in Fig. 6. Hence, the peak value is achieved at $x[0]$ when all active carriers are modulated with S_{HP} . If N_a is the total number of active carriers, then

$$P_{\text{MAX}} = \left(\frac{1}{\sqrt{N}} N_a S_{\text{HP}} \right) \left(\frac{1}{\sqrt{N}} N_a S_{\text{HP}} \right)^* = \frac{2}{N} N_a^2 (\sqrt{M} - 1)^2 \quad (17)$$

$$P_{\text{AVG}} = \frac{E_{\text{AVG}}}{N} = \frac{E_{\text{AVG}}^{\text{QAM}} N_a}{N} = \frac{2N_a(M-1)}{3N} \quad (18)$$

$$\text{PAPR} = \frac{P_{\text{MAX}}}{P_{\text{AVG}}} = \frac{3N_a(\sqrt{M} - 1)^2}{M - 1} = \frac{3N_a(\sqrt{M} - 1)}{\sqrt{M} + 1} \quad (19)$$

Equations (17)-(19) are a generalization of equations (14)-(16) for an arbitrary number of active carriers. The PAPR depends on both the number of active carriers, expressed by N_a , and the way they are modulated, expressed by the ratio $\frac{3(\sqrt{M}-1)}{\sqrt{M}+1}$. The best PAPR is achieved in Frequency Shift Keying (FSK) because $N_a = 1$ and there is no carrier modulation. The worst PAPR is achieved in the case of conventional M -QAM OFDM when $N_a = N$, and both N and M are as high as possible. An advantage of SIM-OFDM over conventional OFDM comes from the fact that in general it has less active carriers. For example, M -QAM SIM-OFDM has half the PAPR when compared to M -QAM OFDM for any number of carriers.

VII. CONCLUSION

A modification of the recently proposed SIM-OFDM was presented in this paper. The new scheme demonstrates improvement over conventional OFDM in terms of energy requirement at low spectral efficiency. It is a potential technique in the design of communication systems with a strong non-linear characteristic where a low PAPR is required such as in optical wireless communications where incoherent light from off-the-shelf light emitting diodes is used to modulate data [6]. An alternative application are uplink channels, where high data rates are not of crucial importance, but energy efficiency. SIM-OFDM can be beneficial in scenarios with a high density of access points, for example in visual light communication uplinks where sophisticated coding schemes require additional design and processing complexity. Further research to investigate SIM-OFDM's performance in comparison to different coding methods and PAPR reduction techniques can fully characterize it as a novel modulation scheme.

ACKNOWLEDGEMENT

We gratefully acknowledge support for this work from the UK Engineering and Physical Sciences Research Council (EPSRC) under grant EP/I013539/1.

REFERENCES

- [1] R. Abualhiga and H. Haas, “Subcarrier-Index Modulation OFDM,” in *Proc. of the International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Tokyo, Japan, Sep. 13–16, 2009.
- [2] D. Grace, J. Chen, T. Jiang, and P. Mitchell, “Using Cognitive Radio to Deliver ‘Green’ Communications,” in *Proc. of the 4th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*. Hannover, Germany: IEEE, Jun. 22-24, 2009, pp. 1–6.
- [3] M.-S. Alouini and A. Goldsmith, “A Unified Approach for Calculating Error Rates Of Linearly Modulated Signals over Generalized Fading Channels,” *IEEE Transactions on Communications*, vol. 47, no. 9, pp. 1324–1334, 1999.
- [4] F. Xiong, *Digital Modulation Techniques*, 2nd ed. Artech House Publishers, 2006.
- [5] R. J. Green, H. Joshi, M. D. Higgins, and M. S. Leeson, “Recent Developments in Indoor Optical Wireless,” *IET Communications*, vol. 2, no. 1, pp. 3–10, Jan. 2008.
- [6] S. Dimitrov and H. Haas, “On the Clipping Noise in an ACO-OFDM Optical Wireless Communication System,” in *IEEE Global Communications Conference (IEEE GLOBECOM 2010)*, Miami, Florida, USA, 6–10 Dec. 2010.