Laminar flow over a steadily rotating circular cylinder under the influence of uniform shear

Sangmo Kang

Citation: Phys. Fluids 18, 047106 (2006); doi: 10.1063/1.2189293
View online: http://dx.doi.org/10.1063/1.2189293
View Table of Contents: http://pof.aip.org/resource/1/PHFLE6/v18/i4
Published by the AIP Publishing LLC.

Additional information on Phys. Fluids
Journal Homepage: http://pof.aip.org/
Journal Information: http://pof.aip.org/about/about_the_journal
Top downloads: http://pof.aip.org/features/most_downloaded
Information for Authors: http://pof.aip.org/authors
Laminar flow over a steadily rotating circular cylinder under the influence of uniform shear

Sangmo Kang\textsuperscript{a)}

Department of Mechanical Engineering, Dong-A University, Busan 604-714, Korea

(Received 31 May 2005; accepted 21 December 2005; published online 19 April 2006)

The present study has numerically investigated two-dimensional laminar flow over a steadily rotating circular cylinder with uniform planar shear, where the free-stream velocity varies linearly across the cylinder diameter. It aims to find the combined effects of shear and rotation on the flow. Numerical simulations using the immersed-boundary method are performed on the flow in the ranges of mainly $|\alpha| \leq 2.5$ (low rotational speeds) and additionally $2.5 < |\alpha| \leq 5.5$ (higher rotational speeds) for $0 \leq K \leq 0.2$ at a fixed Reynolds number of $Re = 100$, where $K$ and $\alpha$ are, respectively, the dimensionless shear rate and rotational speed. For low rotational speeds, results show that the positive shear, with the upper side having the higher free-stream velocity than the lower, favors the effect of the counterclockwise rotation ($\alpha > 0$) but counteracts that of the clockwise rotation ($\alpha < 0$). Accordingly, the absolute critical rotational speed ($|\alpha_c|$), below which von Kármán vortex shedding occurs, decreases with increasing $K$ for $\alpha > 0$, but increases for $\alpha < 0$. The vortex-shedding frequency ($St$) rises with increasing $\alpha$ (including the negative) and the increment slope ($dSt/da$) becomes steeper with increasing $K$. The mean lift slightly decreases with increasing $K$ regardless of the rotational direction. However, the variations of the amplitudes of the lift- and drag-fluctuations as well as the mean drag with the shear rate depend strongly on the rotational direction. They all decrease with increasing $K$ for $\alpha > 0$, but rise for $\alpha < 0$. For higher rotational speeds, on the other hand, the flow characteristics also depend significantly on the shear rate and rotational speed (including the rotational direction). Flow statistics as well as instantaneous flow fields are presented to identify the characteristics of the flow and then to understand the underlying mechanism.

\textcopyright 2006 American Institute of Physics. [DOI: 10.1063/1.2189293]

I. INTRODUCTION

In flow past a bluff body, wall motion has been accepted as one kind of boundary-layer flow controls. When a wall moves downstream, the relative motion between the wall and free stream becomes minimized, thereby inhibiting the growth of the boundary layer. Further, the wall movement injects additional momentum into the boundary layer. Consequently, flow separation is delayed, leading to reduction in the drag, increase in the lift, and suppression of the flow-induced oscillation, to name a few.\textsuperscript{1,2}

Flow past a circular cylinder has, so far, been a subject of numerous investigations as a building-block problem for such flow controls using wall motion in the bluff-body flow. When a circular cylinder immersed in uniform flow rotates at a constant speed on a center axis, the lift force is exerted due to the Magnus effect and its magnitude rises with increasing rotational speed.\textsuperscript{3–8} With the circumferential velocity exceeding approximately two times the free-stream velocity, von Kármán vortex shedding that appears for low rotational speeds completely vanishes and, thus, the flow-induced oscillation does not occur any more. From a practical point of view, however, it is prohibitively complicated to exploit the wall motion by entirely rotating a bluff-body of nonsimple geometry other than a circular cylinder or a sphere. In flow past a bluff body of complex geometry, therefore, the wall-motion effect for controlling vortex shedding has to be achieved in other circuitious ways. Such ways involve utilization of a separate rotating circular cylinder; refer to Gad-El-Hak\textsuperscript{1} and Mittal\textsuperscript{2} for more detailed reviews. According to their reviews, there have been a number of such investigations in which a rotating cylinder could be successfully employed to energize the boundary layer, thereby delaying flow separation orsuppressing vortex shedding. For example, a rotating (control) cylinder could not only be substituted for a small portion of the surface of, say, an airfoil, but also could be properly placed close to a main bluff body.

Quite a few studies have been performed so far on flow past a rotating circular cylinder, but mainly when the free stream is uniform.\textsuperscript{3–8} In most bluff-body flows of engineering interest, however, the free stream is not uniform, but sheared. As evidently observed, air and tidal currents have nonzero velocity gradients due to the formation of boundary layers and, thus, can be regarded as sheared. Such examples involve aircraft in the air, buildings and transport vehicles on the ground, and pipelines under the sea. When a rotating control cylinder is used immediately adjacent to or close to the specified bluff body, the cylinder is inevitably subject to the effect of shear induced from the bluff body. Nevertheless, shear flow or nonuniform flow over a rotating circular cylinder has much less extensively been investigated than the uniform flow. Therefore, more systematic study on the shear flow is required for further improved understanding of such
rotating-cylinder flow. For the study, shear flow can be assumed, as a first approximation, to have a constant lateral velocity gradient, that is a linear velocity profile across the cylinder diameter; refer to Fig. 1.

Uniform-shear flow past a steadily rotating circular cylinder can be regarded as uniform flow past a stationary circular cylinder on which uniform shear and steady rotation are simultaneously imposed. It is remarkable to know, from previous studies, that there is a close correspondence between the two effects of shear and rotation on the flow. When the positive velocity gradient (see Fig. 1), with the upper side having the higher free-stream velocity than the lower, is imposed on uniform flow past a stationary cylinder, the relative motion between the cylinder surface and free stream increases on the upper side (+y direction) about the cylinder, leading to higher shear. On the contrary, the relative motion decreases on the lower side, leading to lower shear. Therefore, the (mean) lift force is exerted from the upper side to the lower, that is from the higher-shear side to the lower-shear, and its magnitude increases linearly in proportion to the velocity gradient.\(^9\)\(^11\) On the other hand, when a circular cylinder immersed in uniform flow rotates steadily in the counterclockwise (positive) direction (see Fig. 1), the relative motion between the cylinder surface and free stream on the upper side becomes higher than that on the lower side. Therefore, the (mean) lift force is also exerted from the upper side with the higher shear to the lower side with the lower shear, and its magnitude increases linearly in proportion to the rotational speed while vortex shedding occurs.\(^5\)\(^7\)

It is a matter of course that the effect of shear on the flow is appreciably small, compared with the rotation effect. Nevertheless, there is obviously a strong resemblance between the two effects; the counterclockwise (clockwise) cylinder rotation and the positive (negative) free-stream velocity-gradient. It is, therefore, expected that, when the shear and rotation are simultaneously applied, the positive shear favors the effect of the counterclockwise rotation but counteracts that of the clockwise rotation. It motivates us to thoroughly reveal the combined effects of shear and rotation on flow past a circular cylinder.

As depicted in Fig. 1, the free stream with a linear velocity profile, \(U = U_c + G y\), passes over a circular cylinder with a diameter, \(D\), and the cylinder rotates at a constant angular velocity, \(\dot{\theta}\), in the counterclockwise direction. Here, \(U_c\) and \(G\) are, respectively, the streamwise velocity at the centerline \((y=0)\) and the lateral velocity gradient \((G = dU/dy)\). Therefore, the flow is governed by the four dimensionless flow parameters: the first one is the Reynolds number, \(Re = U_c D/\nu\), the second is the dimensionless velocity gradient or shear rate, \(K = GD/\nu\), and the third is the dimensionless rotational speed, \(\alpha = \dot{\theta} D/2 U_c\), where \(\nu\) is the kinematic viscosity. In most of previous experimental studies on uniform-shear flow over a (stationary or rotating) cylinder,\(^9\)\(^11\)\(^14\) the lateral width, \(W\), of the flow domain was restricted such that the streamwise velocity in the free stream was constantly positive \((U > 0)\), that is, the free stream could not flow in the reverse direction due to the imposed shear rate. Accordingly, the experimental results had to inevitably entail the blockage effect.\(^11\) Likewise, the present study will also restrict the lateral width to a finite extent, implying that the blockage ratio, defined as \(B = D/W\), becomes the fourth flow parameter governing the flow.

Only a few researches have been performed on uniform-shear flow over a steadily rotating circular cylinder up to now and the typical ones are listed, along with the adopted flow conditions, in Table I. Numerical simulations were performed by Yoshino and Hayashi\(^15\) and Chew et al.,\(^16\) whereas experimental measurements were, to the best of our knowledge, only by Sung et al.\(^14\) They experimentally investigated the flow over the ranges of \(K \lesssim 0.15\), \(-2 \leq \alpha \leq 2\), and \(600 \leq Re \leq 1200\). They claimed that the dominant shedding frequency shifted to a higher value when either \(|\alpha|\) or \(K\) increased. When \(|\alpha|\) increased beyond a certain threshold value, the dominant frequency became less distinct. That is, for \(|\alpha| \gtrsim 1.5\), the velocity field became randomized and diffuse and simultaneously the organized von Kármán vortex street activity got weakened. As reviewed above, there have been too few previous studies considering the combined effects of shear and rotation on the flow.

The objectives of the present study are to numerically investigate the characteristics of two-dimensional laminar flow over a steadily rotating circular cylinder with uniform planar shear, and then to further understand the corresponding underlying mechanism. For the study, we will concentrate on scrutinizing the combined effects of shear rate, rotational speed and blockage ratio on the flow. Numerical simulations are performed using the immersed-boundary method developed by Kim et al.\(^17\) for simulating flows over

---

**TABLE I. Flow conditions used in previous studies.** NA= numerical analysis and EM= experimental measurement.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Flow conditions</th>
<th>Re</th>
<th>(K)</th>
<th>(\alpha)</th>
<th>(B(%))</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yoshino and Hayashi</td>
<td></td>
<td>80</td>
<td>0−0.4</td>
<td>−2−0</td>
<td>=0</td>
<td>NA</td>
</tr>
<tr>
<td>Sung et al. 1995</td>
<td></td>
<td>600−1200</td>
<td>0−0.15</td>
<td>−2−2</td>
<td>10</td>
<td>EM</td>
</tr>
<tr>
<td>Chew et al. 1997</td>
<td></td>
<td>1000</td>
<td>−0.3−0.3</td>
<td>0.5</td>
<td>=0</td>
<td>NA</td>
</tr>
</tbody>
</table>

Reference 15.

\(^1\)Reference 14.

\(^2\)Reference 16.
or inside complex geometries. In the immersed-boundary method, both momentum forcing and mass source/sink are applied on the body surface or inside the body to satisfy the no-slip condition and continuity on or around the immersed boundary, leading to significant memory and CPU savings and easy grid generation compared to the unstructured grid method. In the present study, we will deal with the flow in the ranges of $0 \leq K \leq 0.2$, mainly $|\alpha| \leq 2.5$ (low rotational speeds) and additionally $2.5 < |\alpha| \leq 5.5$ (higher rotational speeds), and $B=0.1$ and 0.05 at a fixed Reynolds number of $Re=100$ that is assumed to be two-dimensional and laminar. Here, the numerical simulations in the range of higher rotational speeds aim to examine the effect of shear rate on the second vortex-shedding mode which occurs at $|\alpha|=5$ in the uniform flow $(K=0).^{6-8}$ This article is organized as follows: Sec. II explains details of the numerical method. Subsequently, in Sec. III, we discuss the results from the simulations. Finally, conclusions are presented in Sec. IV.

II. NUMERICAL METHOD

Numerical simulations of unsteady two-dimensional incompressible flow with uniform planar shear over a steadily rotating circular cylinder are conducted using the immersed-boundary method. The appropriate governing equations can be written as

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_i \partial x_j} + f_i, \quad (1)$$

$$\frac{\partial u_j}{\partial x_i} - q = 0. \quad (2)$$

Here, $x_i$ are the Cartesian coordinates, $u_i$ is the corresponding velocity components, and $p$ is the pressure. All variables are nondimensionalized by the cylinder diameter, $D_c$ and the free-stream velocity at the centerline $(y=0)$, $U_c$. Notice that the notation sets $(u,v)$ and $(x,y)$ are used interchangeably with $(u_1,u_2)$ and $(x_1,x_2)$, respectively. In addition, the notation sets $(r, \theta)$ and $(u_\theta, u_r)$ denote the cylindrical coordinates and their corresponding velocity components, respectively. The discrete-time momentum forcing, $f_i$, is applied to satisfy the no-slip (or rotational) condition on the immersed boundary, whereas the mass source/sink, $q$, is to satisfy the mass conservation for the cell containing the immersed boundary. Therefore, $f_i$ and $q$ are defined, respectively, only at the faces and center of the cell on the immersed boundary or inside the body.

The governing equations, (1) and (2), are integrated in time using a second-order semi-implicit fractional-step method: a third-order Runge-Kutta method for the convection terms and a second-order Crank-Nicolson method for the diffusion terms. In space, on the other hand, the governing equations are resolved with a finite-volume approach on a staggered mesh. Here, the Cartesian (x,y) coordinate system is adopted as a basis for the application of the immersed-boundary method. All spatial derivatives are discretized with the second-order central difference scheme. In addition, a second-order linear or bilinear interpolation scheme is applied to satisfy the no-slip condition on the immersed boundary. More details associated with the immersed-boundary method are described in Kim et al.17

The present computational domain extends to $|x| \leq 40$ and $|y| \leq 1/(2B)$ and the circular cylinder is located with the center at $(0,0)$. The shear rate applicable depending on the blockage ratio is $K \leq 2B$; e.g., $K \leq 0.2$ for $B=0.1$ and $K \leq 0.1$ for $B=0.05$. A Dirichlet boundary condition of the uniform-shear steady flow, $u=1+K y$ and $v=0$, is used at the inflow $(x=-40)$, and a convective outflow condition, $\partial u_i/\partial t + c \partial u_i/\partial x = 0$, is used at the outflow $(x=40)$, where $c$ is the space-averaged streamwise exit velocity. The no-slip or rotational condition, $u_r=0$ and $u_\theta=\alpha$, is imposed at the cylinder surface expressed with $x^2+y^2=0.5^2$. At the far-field boundaries $[y=\pm 1/(2B)]$, on the other hand, two kinds of boundary conditions are attempted for checking their applicability to the present study: a constant-vorticity condition, $\omega=-K$ (or $\partial u_\theta/\partial y=K$) and $v=0$, and a constant-velocity condition, $u_\theta=1/K/(2B)$ and $v=0$. Both the boundary conditions lead to no significant difference in the computational results, and thus we will present only the results achieved from the former condition in this article.

For more efficient simulations, the computational domain is spatially resolved such that a dense clustering of grid points is applied near the cylinder, especially in the wake zone, whereas away from the cylinder a coarser grid is used. In the present study, a uniform distribution of $64 \times 64$ grid points is used within the cylinder diameter, whereas the tangential-hyperbolic grid distribution is in the outer region. As the lateral width of the flow domain changes, the number of total grid points in the y direction is properly adjusted such that the resolution close to the cylinder is preserved. As examples, the spatial resolutions used for $B=0.1$ and 0.05 are, respectively, $M \times N=513 \times 193$ and $513 \times 225$. For time advancement in all the computations, a computational time step is flexibly adjusted such that CFL=$|u_i|/\Delta x_i \max \Delta t=1$.

To confirm the spatial and temporal convergence, parametric studies for $\alpha=1$ and $-1$ at $K=0.2$ and $Re=100$ in the case of $B=0.1$ have been performed and the typical results are presented in Table II. Here, St is the Strouhal number, $S t=f D/U_c$, where $f$ is the vortex-shedding frequency. Moreover, $C_L$ and $C_D$ are, respectively, the time-averaged lift and drag coefficients, whereas $C'_L$ and $C'_D$ are, respectively, the maximum amplitudes of the lift- and drag-coefficient fluctuation...
TABLE III. Validation of the numerical method: comparison study for uniform flow ($K=0$) over a steadily rotating circular cylinder at $Re=100$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\alpha$</th>
<th>$St$</th>
<th>$\bar{C}_L$</th>
<th>$\bar{C}_D$</th>
<th>$C_D$</th>
<th>$C_D'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present ($B=0.0125$)</td>
<td>1.0</td>
<td>0.1654</td>
<td>-2.4787</td>
<td>1.1120</td>
<td>0.3675</td>
<td>0.0997</td>
</tr>
<tr>
<td>Kang et al. (1999)$^a$</td>
<td>1.0</td>
<td>0.1655</td>
<td>-2.4881</td>
<td>1.1040</td>
<td>0.3631</td>
<td>0.0993</td>
</tr>
<tr>
<td>($B=0$)</td>
<td>2.0</td>
<td>-</td>
<td>-5.4327</td>
<td>0.4579</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Stojković et al. (2002)$^b$</td>
<td>1.0</td>
<td>0.1658</td>
<td>-2.504</td>
<td>1.1080</td>
<td>0.3616</td>
<td>0.0986</td>
</tr>
<tr>
<td>($B=0$)</td>
<td>2.0</td>
<td>-</td>
<td>-5.48</td>
<td>0.46</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$^a$Reference 5.  
$^b$Reference 6.

The relative errors in the table show that the results obtained with the chosen parameter values are well converged with respect to the spatial and temporal resolutions. Subsequently, the same numerical simulations have been also performed on uniform flow ($K=0$) over a steadily rotating circular cylinder for $\alpha=1$ and 2 at $Re=100$ (for a very low blockage ratio), and their typical results are compared with other previous ones in Table III. Comparisons in Table III show that the present results are in excellent agreement with those of Kang et al.$^5$ and Stojković et al.$^6$, certainly validating the present immersed-boundary method.

II. RESULTS

After verifying the numerical method, we have conducted numerical simulations by systematically varying the shear rate, rotational speed and blockage ratio in the fairly wide ranges of $0 \leq K \leq 0.2$, mainly $|\alpha| \leq 2.5$ (low rotational speeds) and additionally $2.5 < |\alpha| \leq 5.5$ (higher rotational speeds), and $B=0.1$ and 0.05 at a fixed Reynolds number of $Re=100$. All the simulations are continued until the flow reaches a fully developed state, where all the flow characteristics are analyzed. Detailed discussion on the results for low rotational speeds will be given in the first two subsections. Subsequently, the results for higher rotational speeds will be briefly discussed in the last subsection.

A. Flow statistics

1. Vortex-shedding frequency

Figure 2 shows the variation of the vortex-shedding frequency with the rotational speed according to the shear rate and blockage ratio for the flow at $Re=100$ in the range of low rotational speeds, $|\alpha| \leq 2.5$. For low absolute rotational speeds ($|\alpha|$), vortices of the positive and negative signs are alternately shed, resulting in time-periodic flow, which is called the von Kármán vortex shedding. When the absolute rotational speed exceeds a certain threshold value, von Kármán vortex shedding does not occur any more, leading to steady flow. In other words, vortex shedding occurs over the range of $\alpha_{Lm} \leq |\alpha| \leq \alpha_{Lp}$, whereas otherwise it does not. Here, $\alpha_{Lm}$ ($<0$) and $\alpha_{Lp}$ ($>0$) are the corresponding critical rotational speeds when the cylinder rotates in the clockwise ($\alpha<0$) and counterclockwise ($\alpha>0$) directions, respectively. In the case of no shear ($K=0$), the flow is symmetric with respect to the rotational direction, resulting in the same magnitude of two critical rotational speeds ($|\alpha_{Lm}| = |\alpha_{Lp}|$).

In the case of nonzero shear ($K>0$), on the other hand, the symmetry does not exist any more and, thus, the two absolute critical rotational speeds become different ($|\alpha_{Lm}| \neq |\alpha_{Lp}|$). In the present study, the critical rotational speed is determined by checking the flow state with an increment of $\Delta\alpha=0.05$ for each flow condition ($K$ and $B$). According to the results for the flow at $Re=100$, the critical rotational speed for $K=0$ is predicted to be $|\alpha_{Lm}| = |\alpha_{Lp}| \approx 1.85$ (for a very small blockage ratio). This result agrees well with the predictions, $|\alpha_L| \approx 1.8$, of Kang et al.$^5$ and Stojković et al.$^6$.

In the case of no shear ($K=0$), the vortex-shedding frequency hardly changes even for increments in the rotational speed while vortex shedding occurs ($|\alpha| \leq |\alpha_{Lm}| = |\alpha_{Lp}|$): see Fig. 2(b). In other words, the shedding frequency remains nearly constant for low rotational speeds ($|\alpha| \leq 1$), then slightly decreases as $|\alpha|$ approaches $|\alpha_{Lm}|$, and finally drops to zero at $|\alpha| = |\alpha_{Lm}|$. This variation behavior is in good agreement with the numerical results of Hu et al. ($Re=60$),$^4$ Kang et al. ($Re=60–160$),$^5$ Mittal and Kumar ($Re=200$),$^7$ and Stojković et al. ($Re=100$),$^6$ see Fig. 2(b). On the contrary, it substantially differs from the experimental ones of Diaz et al. ($Re=9000$),$^{18}$ Kimura and Tsutahara ($Re=280$ and 370),$^{19}$ and Sung et al. ($Re=900$)$^{14}$ which claimed that the vortex-shedding frequency rose with increasing rotational speed. Mittal and Kumar$^7$ suggested that the discrepancy between the numerical and experimental studies should result from the mutual interaction between the vortex shed-
TABLE IV. Critical rotational speeds, \( \alpha_{Lm} \) and \( \alpha_{Lp} \), in the range of \( |\alpha| \leq 2.5 \) according to \( K \) and \( B \) for the flow at \( Re=100 \). This shows that von Kármán vortex shedding occurs for \( \alpha_{Lm} \leq \alpha \leq \alpha_{Lp} \) whereas otherwise it does not.

<table>
<thead>
<tr>
<th>( B )</th>
<th>( K )</th>
<th>( \alpha_{Lm} )</th>
<th>( \alpha_{Lp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0125</td>
<td>0</td>
<td>-1.85</td>
<td>1.85</td>
</tr>
<tr>
<td>0.05</td>
<td>0</td>
<td>-1.85</td>
<td>1.85</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>-1.95</td>
<td>1.70</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>-1.90</td>
<td>1.90</td>
</tr>
<tr>
<td>0.2</td>
<td>0.1</td>
<td>-2.00</td>
<td>1.75</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>-2.05</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Figure 2(a) shows the variation of the vortex-shedding frequency with the rotational speed while von Kármán vortex shedding occurs \( (\alpha_{Lm} \leq \alpha \leq \alpha_{Lp}) \). For \( K > 0 \), the vortex-shedding frequency \( \left( \frac{St}{\alpha} \right) \) increases linearly in proportion to the absolute rotational speed \( \left( |\alpha| \right) \) for the counterclockwise rotation. On the contrary, the shedding frequency decreases linearly in proportion to the absolute rotational speed for the clockwise rotation. Collectively speaking, the shedding frequency increases linearly in proportion to the rotational speed (including the negative shear). In addition, the increment slope, \( dSt/d\alpha \), notably rises with increasing shear rate. The vortex-shedding frequency remains nearly constant or slightly decreases near the critical rotational speeds, \( \alpha_{Lm} \) and \( \alpha_{Lp} \). Sung et al.\(^8\) claimed by their experiments \((K=0.1 \text{ and } Re=900)\) that, in the case of nonzero shear, the shedding frequency rose with increasing rotational speed \( \left( |\alpha| \right) \) regardless of the rotational direction. However, it obviously differs from the present result. The discrepancy may be attributed to the mutual interaction between the vortex shedding and centrifugal instability due to the three-dimensional effect, as asserted by Mittal and Kumar.\(^7\)

The blockage effect on the vortex-shedding frequency is comparatively strong, too. With increasing blockage ratio at a constant shear rate, the shedding frequency increases uniformly all over the range of the rotational speed while the variation slope, \( dSt/d\alpha \), keeps nearly constant. For example, when the blockage ratio rises from \( B=0.05 \) to \( 0.1 \), the shedding frequency increases as much as \( \Delta St \approx 0.01 \) regardless of the rotational speed.


FIG. 3. (a) Variation of \( C_L \) with \( \alpha \) in the range of \( |\alpha| \leq 2.5 \) according to \( K \) and \( B \) for the flow at \( Re=100 \). (b) Comparison of the present result \((B=0.0125)\) with the previous published data in the case of \( K=0 \) and \( Re=100 \) [\( Re=200 \) for Mittal and Kumar (Ref. 7)].

2. Lift and drag coefficients

Figure 3 shows the variation of the time-averaged lift coefficient, \( C_L \), with the rotational speed with respect to the shear rate and blockage ratio for the flow at \( Re=100 \) in the range of low rotational speeds, \( |\alpha| \leq 2.5 \). According to Fig. 3(b), in the case of no shear \((K=0)\), the mean lift decreases linearly in proportion to the rotational speed while von Kármán vortex shedding occurs \( \left( |\alpha| \leq 1.85 \right) \). In addition, the mean lift is exerted from the upper (lower) side about the cylinder to the lower (upper) for the counterclockwise (clockwise) rotation. It exactly agrees with the results of Kang et al.,\(^5\) Stojković et al.,\(^6\) and Mittal and Kumar;\(^7\) see Fig. 3(b). The latter two research groups\(^6,7\) observed that the variation slope of the mean lift with the rotational speed, \( dC_L/d\alpha \), remarkably changed at \( |\alpha| \approx 2 \). Especially, Stojković et al.\(^6\) claimed that the mean lift no longer increased linearly but showed a progressive increase with the rotational speed which could be fitted by a quadratic relation of the form \( C_L = a + b \cdot \alpha^2 \) \((a \approx 1.322, b \approx 0.994)\) for higher rotational speeds \((2 \leq \alpha \leq 5)\). Their results are also in good agreement with the present one shown in Fig. 3(b); see also Fig. 12(a) in advance.

In the case of nonzero shear \((K > 0)\), on the other hand, the linear relation between the mean lift and rotational speed hardly changes even for alterations in the shear rate and blockage ratio while vortex shedding occurs; see Fig. 3(a). It is due to the fact that since the absolute value of the lift coefficient is very large compared to the drag coefficient the relative variation of the lift coefficient with the shear rate and blockage ratio becomes negligibly small. To more closely see
the effect of shear rate and blockage ratio on the mean lift, the least-square fit has been applied for the range of \(|\alpha| \leq 1\) with the assumption of a linear relation between the mean lift and rotational speed, and then the typical results are presented in Table V. When the cylinder is stationary \((\alpha=0)\), the mean lift decreases (that the absolute value increases in the negative direction) with increasing shear rate, whereas it rises with increasing blockage ratio. When the cylinder rotates \((\alpha \neq 0)\), in the other respect, the absolute variation slope of the mean lift with the rotational speed, \(|d\bar{C}_L/d\alpha|\), decreases with increasing shear rate, whereas it rises with increasing blockage ratio. It is also seen that the variation slope remarkably changes near the critical rotational speeds \((|\alpha| \approx 1.5–2)\).

Figure 4 shows the variation of the mean drag coefficient, \(\bar{C}_D\), with the shear rate, rotational speed and blockage ratio. In the case of no shear \((K=0)\), the mean drag attains a maximum value at \(\alpha=0\) and then gradually decreases with increasing rotational speed \((|\alpha|)\). This variation behavior is in exact agreement with the numerical results of Kang et al.,\(^5\) Stojković et al.\(^6\) and Mittal and Kumar;\(^7\) see Fig. 4(b). In the case of nonzero shear \((K>0)\), on the other hand, the rotational speed for the maximum lift shifts in the negative direction as the shear rate increases: see Fig. 4(a). In addition, with increasing shear rate, the mean drag decreases for the counterclockwise rotation \((\alpha \approx -0.5)\), whereas it increases for the clockwise rotation \((\alpha \approx -0.5)\), although the former case has larger variation \((|d\bar{C}_D/d\alpha|)\) than the latter case at the same absolute rotational speed. Meanwhile, the mean drag increases, regardless of the rotational direction and speed, with increasing blockage ratio at a constant shear rate. The variation slope of the mean drag with the rotational speed, \(|d\bar{C}_D/d\alpha|\), also drastically changes near the critical rotational speeds \((|\alpha| \approx 1.5–2)\) like for the mean lift.

Figure 5 shows the variation of the maximum amplitude of the lift-coefficient fluctuations, \(C_L^\text{rms}\), with the shear rate, rotational speed and blockage ratio for the flow at \(Re=100\) in the range of low rotational speeds, \(|\alpha| \leq 2.5\). In the case of no shear \((K=0)\), with increasing rotational speed \((|\alpha|)\), the lift fluctuation gradually rises for low rotational speeds, attains a maximum value at \(\alpha=1\) and then drops near the critical rotational speed \((|\alpha| \approx 1.85)\). The present variation behavior is in exact agreement with those of Kang et al.,\(^5\) Stojković et al.\(^6\) and Mittal and Kumar;\(^7\) see Fig. 5(b). In the case of nonzero shear \((K>0)\), on the other hand, the fluctuation characteristics significantly depend on the rotational speed.
direction: see Fig. 5(a). With increasing shear rate, the lift fluctuation slightly decreases for the counterclockwise rotation, whereas it largely increases for the clockwise rotation. As a result, the lift fluctuation rather decreases with increasing absolute rotational speed for the counterclockwise rotation at relatively large shear rates (K ≥ 0.1) unlike the no-shear case (K = 0). On the contrary, the lift fluctuation rises with increasing absolute rotational speed at low rotational speeds for the clockwise rotation and the variation slope, |dC_f/da|, rises with increasing shear rate. However, the blockage effect hardly depends on the rotational direction. In other words, the lift fluctuation slightly rises, regardless of the rotational direction, with increasing blockage ratio.

Figure 6 shows the variation of the maximum amplitude of the drag-coefficient fluctuations, C_d, with the shear rate, rotational speed and blockage ratio. Here, Figs. 5 and 6 show close affinities between the characteristics in the lift and drag fluctuations. In the case of no shear (K = 0), the drag fluctuation increases linearly in proportion to the rotational speed for low rotational speeds and then drastically decreases for |α| ≥ 1.5. This variation behavior agrees exactly with those of Kang et al., Stojković et al., and Mittal and Kumar: see Fig. 6(b). In the case of nonzero shear (K > 0), on the other hand, the characteristic in the drag fluctuation significantly depends on the rotational direction: see Fig. 6(a). With increasing shear rate, the drag fluctuation slightly decreases for the counterclockwise rotation, whereas it largely increases for the clockwise rotation. In addition, with increasing blockage ratio, the drag fluctuation increases regardless of the rotational direction. It indicates that, when the signs of the shear rate and cylinder rotation are identical (for example the counterclockwise rotation and positive velocity gradient), the activity of shed vortices becomes weakened and, thus, the fluctuation amplitude decreases. When the signs are different, on the contrary, the activity becomes strengthened and, thus, the fluctuation amplitude increases.

3. Mean flow quantities

Figures 7 and 8 show distributions of the mean pressure coefficient, C_p, and the mean vorticity, ɷ, on the cylinder surface according to the shear rate and rotational speed for the flow at Re=100 and B=0.1 in the range of low rotational speeds, |α| ≤ 2.5. The mean flow quantities are obtained by time-averaging the corresponding quantities over one complete period after the flow becomes fully developed.

It is well known that, in flow over a bluff body such as a circular cylinder, the effect of friction on the mean lift force is negligibly small compared to the pressure. Therefore, to further understand the variation of the mean lift with the shear rate and rotational speed, it is essential to exactly know the mean-pressure distribution around the cylinder surface. Figure 7 shows the mean pressure coefficient. Here, the pressure coefficient is defined as 
\[ C_p = 2(p - p_c)/\rho U_c^2, \]
where p_c is the pressure corresponding to the free-stream velocity, U_c, at the centerline (y=0) far away from the cylinder. In the case of no shear (K = 0), the mean pressure for no rotation (α=0) is symmetric about θ=0−180° (base and stagnation points), leading to a zero mean lift. As the rotational speed increases in the counterclockwise direction (α>0), the surface pressure on the lower side (near θ=270°) drastically decreases, causing the increase in the mean lift exerted toward the lower side. The result is in exact agreement with the
numerical ones of Kang et al.⁵ and Mittal and Kumar.⁷ For no rotation, with increasing shear rate, the mean pressure remains nearly constant in the range of \( \theta \approx 180° - 240° \) but increases uniformly all over the cylinder surface except for the range. Therefore, the simple analysis of force balance indicates that the increase in the shear rate raises little by little the mean lift in the negative (\( -y \)) direction.

From the pressure distribution on the cylinder surface, however, it is obvious that the nonzero shear rate (\( K > 0 \)) has different effects on the flow according to the rotational direction when the cylinder rotates (\( \alpha \neq 0 \)). For the counterclockwise rotation (\( \alpha > 0 \)), the mean pressure increases uniformly in the ranges of \( \theta \approx 0° - 180° \) and \( \theta \approx 240° - 360° \) with increasing shear rate. Considering the simple force balance, the mean lift is exerted in the \( -y \) direction and its magnitude increases with increasing shear rate. For the clockwise rotation (\( \alpha < 0 \)), on the other hand, the mean pressure slightly increases in the range of \( \theta \approx 90° - 180° \) but largely decreases in the range of \( \theta \approx 180° - 360° \). Thus, the mean lift is exerted in the \( +y \) direction and its magnitude decreases with increasing shear rate. As a whole, however, the dependency of the shear rate on the mean surface pressure and, thus, the mean lift for the counterclockwise rotation is stronger than the opposite case.

Figure 8 shows distribution of the mean vorticity on the cylinder surface according to the shear rate and rotational speed. In the case of no shear (\( K = 0 \)), for no rotation (\( \alpha = 0 \)), the mean vorticity has a positive peak and a negative peak, respectively, at \( \theta \approx 230° \) and \( \theta \approx 130° \) and is nearly zero in the wake zone. Subsequently, with increasing rotational speed in the counterclockwise direction (\( \alpha > 0 \)), the positive peak value increases but the negative one decreases in its magnitude. In the wake zone (\(-60° \leq \theta \leq 80°\)), the mean vorticity is all negative and simultaneously new positive and negative peaks are formed. Moreover, with increasing absolute rotational speed (\( |\alpha| \)), the locations for the new positive and negative peaks shift in the rotational direction. It agrees well with the results of Kang et al.⁵ and Mittal and Kumar.⁷

In the case of nonzero shear (\( K > 0 \)), many changes are observed near the positive and negative peaks of the mean vorticity when the cylinder rotates. With increasing shear rate, the mean vorticity near the positive peak (\( 210° \leq \theta \leq 300° \)) decreases without regard to the rotational direction, while the vorticity near the negative peak (\( 60° \leq \theta \leq 150° \)) increases in its magnitude. In the range of \( 150° \leq \theta \leq 210° \), on the other hand, the mean vorticity uniformly moves toward the positive vorticity value (\( \omega > 0 \)). That is, the absolute value of the positive vorticity increases in the range, whereas that of the negative one decreases.

### B. Flow dynamics

Instantaneous flow fields have been investigated to understand vortex shedding and its corresponding underlying mechanism according to the shear rate and rotational speed in uniform-shear flow over a steadily rotating circular cylinder for the range of low rotational speeds, \( |\alpha| \leq 2.5 \). Results show that, although the flow statistics depend strongly on the blockage ratio as shown in Figs. 2–6, the appearance of the instantaneous flow field does not vary so significantly. In this article, therefore, only the results for the flow at \( Re = 100 \) and \( B = 0.1 \) will be presented.

Figure 9 shows the variation of the instantaneous vorticity contours with the rotational speed for the flow at \( K = 0.2 \) (\( Re = 100 \) and \( B = 0.1 \)). In Fig. 9, the vorticity contours at the time of the maximum lift force in one complete period are presented for each rotational speed. Note that the background vorticity in each case is \( \omega = -K \) and thus negative. According to Table IV, von Kármán vortex shedding occurs in the range of \( -2.05 \leq \alpha \leq 1.60 \) for the flow at \( K = 0.2 \) and \( B = 0.1 \), whereas out of the range it completely disappears, leading to steady flow. Such an observation is confirmed again by the vorticity contours shown in Fig. 9. Especially, Figs. 9(b) and 9(f) correspond to the two critical rotational speeds, \( \alpha_{Lm} \) and \( \alpha_{Lp} \), respectively.

In the case of no rotation (\( \alpha = 0 \)), vortices of the positive and negative signs are alternately shed and then they move downstream [refer to Fig. 9(d)]. However, the positive shear rate (\( K = 0.2 \)) imposed on the free stream makes the flow asymmetric about the cylinder in the sense of time-averaging. Due to the negative background vorticity (\( \omega = -0.2 \)), the negative-signed vortices in the wake become strengthened and round-shaped, whereas the positive-signed vortices become weakened and elongate-shaped.

With increasing rotational speed in the counterclockwise direction (\( \alpha > 0 \)), all the vortices become narrower compared to the case of no rotation, leading to decrease in the lateral width of wake [refer to Fig. 9(e)]. For the clockwise rotation (\( \alpha < 0 \)), on the contrary, all the vortices become thicker, leading to increase in the width of wake [refer to Fig. 9(c)].
Such a phenomenon (the wake width according to the rotational direction) can also be confirmed by the vorticity contours for the two steady states at \( \alpha = \pm 2.5 \), where the wake width for the clockwise rotation is much larger than the opposite case. As a whole, the wake width decreases with increasing rotational speed in the counterclockwise direction while vortex shedding occurs, whereas the wake width increases in the clockwise direction. Sumner et al.,\textsuperscript{10} Sumner and Akosile,\textsuperscript{11} and Mittal and Kumar\textsuperscript{7} claimed that, in flow over one or multiple circular cylinder(s), the decrease in the lateral width of the shed vortices, that is, wake, lead to increase in the vortex-shedding frequency while vortex shedding occurred. Their results exactly agree with the present one. In other words, Figs. 9 and 2 clearly indicate that the wake width decreases with increasing rotational speed (including the negative) and simultaneously the vortex-shedding frequency increases.

To scrutinize the effect of shear on the wake when the cylinder rotates, Fig. 10 shows the variation of the instantaneous vorticity contours with the shear rate for the flow at \( \alpha = -1.5 \) and 1.0. In the case of no shear \((K=0)\), the positive-signed vortex for the clockwise rotation becomes more elongate-shaped compared to the counterclockwise rotation, whereas the negative-signed vortex becomes more round-shaped. In the case of nonzero shear \((K > 0)\), on the other hand, with increasing shear rate, the negative-signed vortex in the near wake becomes more strengthened and more round-shaped all over the rotational speeds due to the negative background vorticity. On the contrary, the positive-signed vortex becomes more weakened and more elongate-shaped. Moreover, with traveling downstream, the distance between two successive positive-signed vortices becomes narrower than that for the negative-signed vortices due to the difference in the free-stream velocity. Simultaneously, the vortices shift in the clockwise direction because of the negative background vorticity. However, with changing shear rate, the lateral wake width, which closely relates to the vortex-shedding frequency,\textsuperscript{7,11,20} varies in different ways depending on the rotational direction. In other words, the wake width rises with increasing shear rate for the clockwise rotation, leading to decrease in the vortex-shedding frequency. For the counterclockwise rotation, on the contrary, the wake width decreases and thus the shedding frequency increases.

C. On higher rotational speeds

Recently, Stojkovi\'\v{c} et al. (Re=100),\textsuperscript{5} Mittal and Kumar (Re=200),\textsuperscript{7} and Stojkovi\'\v{c} et al. (60 \leq \text{Re} \leq 200)\textsuperscript{8} reported that a second vortex shedding occurred for higher rotational speeds, \( |\alpha| \approx 5 \), in uniform flow \((K=0)\) over a steadily rotating circular cylinder at low Reynolds numbers. To see the effect of shear rate on the second vortex shedding, the same numerical simulations have also been performed on the flow at Re=100 and \( B=0.1 \) in the ranges of \( 2.5 < |\alpha| \leq 5.5 \) (higher rotational speeds) and \( 0 \leq K \leq 0.2 \).

Results show that the second vortex shedding indeed occurs in certain ranges of rotational speed at \( |\alpha| = 5 \) and the shedding ranges strongly depend on the shear rate. The rotational-speed bounds, or critical rotational speeds \((\alpha_{1} \text{ and } \alpha_{2})\), for the second vortex shedding according to the shear rate are shown in Table VI. In the case of no shear \((K=0)\), a second vortex shedding occurs in the range of \( 4.8 \leq |\alpha| \leq 5.1 \) at \( B=0.1 \) (for reference, 4.85 \leq |\alpha| \leq 5.2 \) at \( B=0.0125 \) .
The result is in good agreement with those of Stojković et al.\(^6\) (4.8 \(\leq \alpha \leq 5.15\)) and Stojković et al.\(^8\) (4.8 \(\leq \alpha \leq 5.1\)) for the flow at Re=100 in spite of difference in the computational conditions. In the case of nonzero shear (\(K > 0\)), on the other hand, the shedding ranges vary greatly with changing shear rate and the variation behavior depends strongly on the rotational direction.

![Instantaneous vorticity contours at the time of the maximum lift according to K and \(\alpha\) for the flow at Re=100 (\(B=0.1\)): \(\alpha = (a) -1.5\) and (b) 1.]

\[\text{FIG. 10.} \]

Figure 11 shows the variation of the vortex-shedding frequency with the rotational speed in the range of higher rotational speeds, 2.5 \(\leq |\alpha| \leq 5.5\), according to the shear rate. It is found that the vortex-shedding frequency for higher rotational speeds is about one order of magnitude lower than the typical frequency of von Kármán vortex shedding for low rotational speeds, |\(\alpha| \leq 2\): see also Fig. 2 for comparison. It implies the fundamental difference between the two vortex-shedding phenomena. As shown in Fig. 11, in the case of no shear (\(K=0\)), the shedding frequency decreases greatly with increasing rotational speed. The variation behavior, as well as the values of the shedding frequency, agrees well with that of Stojković et al.\(^8\) indicating that the present study can quite exactly reproduce the second vortex-shedding mode for higher rotational speeds.

In the case of nonzero shear (\(K > 0\)), the shedding frequency decreases greatly with increasing absolute rotational speed (|\(\alpha|\)) regardless of the shear rate for the counterclockwise (positive) rotation, which is similar to the case of no shear (\(K=0\)). In other words, the rotational-speed range for the second vortex shedding rises with increasing shear rate, but the increment slope, dSt/d|\(\alpha|\), hardly vary. For the clockwise (negative) rotation, on the other hand, the variation of the shedding frequency with the rotational speed depends on whether the shear rate is low or high. The shedding frequency decreases greatly with increasing absolute rotational speed at low shear rates, 0 \(\leq K \leq K_{cv}\) (the critical value, \(K_{cv}\) in the regime of the second vortex shedding should be between \(K=0.05\) and 0.1 from the present numerical simulations), whereas it increases gradually at higher shear rates, \(K_{cv} < K \leq 0.2\). In addition, with increasing shear rate, the absolute rotational speeds (|\(\alpha|\)) for the second vor-

![Variation of St with \(\alpha\) in the range of 2.5 \(\leq |\alpha| \leq 5.5\) according to K for the flow at Re=100 and B=0.1, compared with the result of Stojković et al. [2003 (Ref. 6)] (presented by the dotted line).]

\[\text{FIG. 11.} \]
higher rotational speeds according to K

flow only for low rotational speeds

increases greatly at higher shear rates.

FIG. 12. Variations of $\bar{C}_L$ and $\bar{C}_D$ with $\alpha$ in the range of $2.5 \leq |\alpha| \leq 5.5$ according to $K$ for the flow at $Re=100$ and $B=0.1$: (a) $\bar{C}_L$ and (b) $\bar{C}_D$.

tex shedding decrease gradually at low shear rates, increase radically across the critical shear rate, $K=K_{cv}$, and then decrease greatly at higher shear rates.

Figure 12 shows the variations of the mean lift and drag coefficients, $\bar{C}_L$ and $\bar{C}_D$, with the rotational speed in the range of $2.5 \leq |\alpha| \leq 5.5$, whereas Fig. 13 shows the variations of the maximum amplitudes of the lift- and drag-coefficient fluctuations, $C'_L$ and $C'_D$. These figures confirm that the shear rate does not have significant effects on the flow only for low rotational speeds ($|\alpha| \leq 2.5$), but also for higher rotational speeds ($2.5 < |\alpha| \leq 5.5$). Further, the variation of the flow characteristics with the rotational speed according to the shear rate for the clockwise rotation substantially differs from that for the counterclockwise rotation. As expected from the variation of the shedding frequency shown in Fig. 11, the flow characteristics for the clockwise rotation vary with the shear rate in more complicated ways than those for the counterclockwise rotation.

The variations of the mean lift and drag coefficients, $\bar{C}_L$ and $\bar{C}_D$, with the rotational speed are shown in Fig. 12. The mean lift rises in its magnitude with increasing absolute rotational speed ($|\alpha|$) regardless of the rotational direction. However, since the (absolute) mean lift is much larger than the mean drag, the relative variation of the mean lift with the shear rate is not so conspicuous. In the other respect, the mean drag depends significantly on the shear rate and rotational direction. For the counterclockwise rotation, the mean drag and its decline slope, $\bar{C}_D$ and $|d\bar{C}_D/d\alpha|$, decrease with increasing rotational speed while no vortex shedding occurs ($2.5 < \alpha < 4.8$), whereas the mean drag rises with increasing shear rate. Reversely, the mean drag rises with increasing rotational speed while the second vortex shedding occurs, but it decreases with increasing shear rate. For the clockwise rotation, on the other hand, the mean drag decreases with increasing absolute rotational speed ($|\alpha|$) while no vortex shedding occurs. However, the decline slope, $|d\bar{C}_D/d\alpha|$, depends on the magnitude of the shear rate. The mean drag decreases greatly with increasing absolute rotational speed at low shear rates, $0 < K < K_{cs}$ (the critical value, $K_{cs}$, in the regime of no vortex shedding should be between $K=0.1$ and 0.2 from the present numerical simulations), whereas it gradually decreases or remains nearly constant at higher shear rates, $K_{cs} < K < 0.2$. In addition, the mean drag decreases with increasing shear rate at low shear rates. During the second vortex shedding, the mean drag increases gradually towards $\bar{C}_D=0$ with increasing absolute rotational speed regardless of the shear rate at low shear rates, whereas the mean drag remains nearly constant at $\bar{C}_D=0$ at higher shear rates [see the case of $K=0.2$ in Fig. 12(b)].

The variations of the maximum amplitudes of the lift- and drag-coefficient fluctuations, $C'_L$ and $C'_D$, are shown in Fig. 13. Results indicate that the lift and drag fluctuations for the second vortex shedding are much larger than those for von Kármán vortex shedding. For the counterclockwise rotation, both the lift and drag fluctuations decrease with increasing shear rate during the second vortex shedding. For the clockwise rotation, on the other hand, the fluctuation characteristics also depend on the magnitude of the shear rate. The lift and drag fluctuations decrease with increasing shear rate at low shear rates, $0 < K < K_{cv}$, rise radically across the critical shear rate, $K=K_{cv}$, and then decrease at higher shear rates, $K_{cv} < K < 0.2$.

IV. CONCLUSIONS

In the present study, we have numerically investigated two-dimensional laminar flow over a steadily rotating circular cylinder with uniform planar shear, where the free-stream velocity varies linearly across the cylinder diameter, to identify the hydrodynamic force and wake dynamics and then to
understand the underlying mechanism. For the study, numerical simulations using the immersed boundary method were performed on the flow in the ranges of 0 ≤ K ≤ 0.2, mainly |α| ≥ 2.5 (low rotational speeds) and additionally 2.5 < |α| ≤ 5.5 (higher rotational speeds), and B=0.1 and 0.05 at a fixed Reynolds number of Re=100. Here, K and α are, respectively, the dimensionless shear rate and rotational speed, whereas B and Re are, respectively, the blockage ratio and Reynolds number. Note that the positive shear rate corresponds to the upper side having higher free-stream velocity than the lower, and the positive rotation denotes a counterclockwise one. Conclusions drawn in the present study can be summarized as follows:

1. Vortex shedding occurred for low rotational speeds, that is in the range of \( \alpha_{Lm} \leq \alpha \leq \alpha_{Lp} \), whereas otherwise it completely disappeared. Here, \( \alpha_{Lm}(<0) \) and \( \alpha_{Lp}(>0) \) are the critical rotational speeds, respectively, for the counterclockwise and clockwise rotations. In the case of no shear (K=0), the two (absolute) critical speeds became identical (\( \alpha_{Lm} = \alpha_{Lp} \approx 1.85 \)). With increasing shear rate, however, \( \alpha_{Lp} \) decreased whereas \( \alpha_{Lm} \) increased, implying that the positive shear (K > 0) favored the effect of the counterclockwise rotation but counteracted that of the clockwise rotation.

2. The vortex-shedding frequency, St, increased linearly in approximate proportion to the rotational speed while vortex shedding occurred (\( \alpha_{Lm} < \alpha \leq \alpha_{Lp} \)). In addition, the variation slope, dSt/dα, markedly rose with increasing shear rate.

3. With increasing shear rate, the mean lift slightly increased without regard to the rotational direction. However, the effect of shear on the mean drag depended on the rotational direction. That is, with increasing shear rate, the mean drag decreased for the counterclockwise rotation (α ≥ -0.5) whereas it slightly rose for the clockwise rotation (α ≤ -0.5).

4. The characteristics in the lift and drag fluctuations according to the shear rate strongly depended on the rotational direction. With increasing shear rate, the lift and drag fluctuations slightly decreased for the counterclockwise rotation (α ≥ 0.5) whereas they greatly rose for the clockwise rotation (α ≤ 0.5). In other words, when the signs of the shear and rotation were equal the activity of vortex shedding became weakened and thus the fluctuations decreased. On the contrary, when the signs were different, the activity became strengthened and the fluctuations much increased.

5. In the case of nonzero shear (K > 0), the lateral width in the wake gradually decreased with increasing rotational speed (|α|) in the counterclockwise direction. For the clockwise rotation, on the contrary, the wake width increased. It was closely related to the observation that the shedding frequency augmented when the rotational speed increased (including the negative).

6. A second vortex shedding occurred at higher rotational speeds, \(|\alpha| \approx 5\), and the shedding frequency was about one order of magnitude lower than the typical frequency of von Kármán vortex shedding for low rotational speeds, \(|\alpha| \leq 2\). The flow statistics depended significantly on the shear rate and rotational speed (including the rotational direction) while the second vortex shedding occurred.

ACKNOWLEDGMENT

This work was supported by the NRL (National Research Laboratory) Program of the Ministry of Science and Technology, Korea.