Research papers

The dynamics of an energetic tidal channel, the Pentland Firth, Scotland

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1.1. Hydrodynamic setting

The dynamics of the shelf tides along the north-west European continental shelf have been studied extensively and can be described with reference to mapped tidal charts (Pugh, 1987; Howarth and Pugh, 1983; Huntley, 1980; Cartwright et al., 1980). Tidal charts show locations where high water occurs simultaneously (co-tidal lines) with contours also joining equal tidal ranges (co-amplitude lines). In the North Atlantic, the largest components in both the ocean and on the shelf are the lunar semidiurnal tide (Howarth and Pugh, 1983). The Atlantic system propagates as a Kelvin wave, anticlockwise (south-to-north) around its amphidromic point, up the west coast of the British Isles. The northward sweep of the Atlantic wave is further influenced by a second amphidromic system located near the Faroe Islands. The Atlantic tide swings eastwards North of the Shetland Isles and into northern reaches of the North Sea.

The tides in the North Sea are also primarily semidiurnal. Proudman and Doodson (1924) produced charts of the spatial distribution of the lunar semidiurnal tide in the North Sea based on short-term current speed measurements. This established the existence of two complete amphidromic systems in the southern North Sea and a third, potentially degenerate, amphidrome around the Southern Tip of Norway. Huthnance (1991) explains that, in the North Sea, the semidiurnal tide progresses cyclonically (anticlockwise) with the largest amplitudes occurring as the Kelvin wave proceeds south along the eastern coast of the United Kingdom.

Separating mainland Scotland from the Orkney Islands, the Pentland Firth is a narrow sea corridor that connects the North Atlantic Ocean to the North Sea (Fig. 1). Analysis of the published charts of the shelf tides around the United Kingdom reveals a distinct difference between the timings of high and low water at either end of the Pentland Firth. On the Atlantic Side, the

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Dillon and Woolf (2008) also identify a difference between wave in the vicinity of the northern Isles as explained above and North Sea side. This is due to the swing of the Atlantic Kelvin approximately 2-h in advance of the same wave travelling south on the northward sweep of the semidiurnal Kelvin wave arrives approximately 2-h in advance of the same wave travelling south on the North Sea side. This is due to the swing of the Atlantic Kelvin wave in the vicinity of the northern Isles as explained above and is indicated by a phase difference of 50–60° between co-tidal lines. Dillon and Woolf (2008) also identify a difference between the respective tidal ranges, with the Atlantic tidal amplitudes larger than those of North Sea. In this context, Mackay (2007) provides an analogy of the Atlantic Ocean acting as a giant piston providing work to the North Sea using the Pentland Firth as an oceanographic connecting rod.

The tidal stream velocities that occur in the Pentland Firth are some of the fastest and most persistent in the world. In recent years, the technological feasibility of harnessing the power of these tidal streams for large-scale commercial electricity generation has become established. As a result, the Pentland Firth is scheduled to host up to 800 MW of installed tidal stream energy capacity, shared over four sites, by the year 2020. The ultimate capacity is expected to be larger than these initial developments.

1.2. Tidal energy resource characteristics

Understanding the hydrodynamic conditions that generate the powerful tidal streams of the Pentland Firth is a prerequisite to effectively and sustainably harnessing the available energy. Couch and Bryden (2006) identify a set of generic hydrodynamic regimes in which tidal stream velocities may be sufficiently accelerated for economical tidal stream power.

Resonant Systems are the amplification of the tide when it is forced at (or near to) its natural period of oscillation (Pugh, 1987). Such a condition may occur in an enclosed basin with constructive interference between the propagating and reflected tidal wave. An example of this phenomenon occurs in the Bay of Fundy, Canada (Garrett, 1972; Godin, 1993; Karsten et al., 2008).

Tidal Streaming is the physical response of the tide to the maintenance of the continuity equation. As the tidal wave propagates onto the shelf, conservation of energy dictates that decreasing water depths are accompanied by increasing wave amplitude and larger tidal stream velocities. For unconstrained coastal areas, however, tidal currents generally remain too weak for economically viable tidal stream energy generation. When the tidal wave is constrained further by coastal geography, such as between two islands, conservation of mass may accelerate the flow to an economically viable tidal stream velocity.

Hydraulic Currents can be distinguished from propagating tidal currents as they are driven by a pressure head developed across a channel due to a difference in tidal elevation at either end (Pugh, 1987). This may be the result of a difference in tidal phase, a difference in tidal range, or a combination of both factors.

Based on these hydrodynamic resource categorisations and the description of the shelf tides around the UK, it would appear appropriate to treat the Pentland Firth as a hydraulic current. The pressure head developed across the channel induces acceleration of the flow and transfer of potential to kinetic energy. Black and Veatch Ltd. (2011) adopt such an approach to determine the extractable energy resource. It is reasonable to expect, however, that the tides within the Pentland Firth will differ from the idealised hydraulic current. The channel contains a number of islands and headlands, plus a sharp bend at the mouth to the North Sea. This will give contributing accelerations from other mechanisms, particularly tidal streaming. Owen and Bryden (2007) indicate that funnelling within the Pentland Firth accelerates the flow to charted values in excess of 6 m s⁻¹. Dillon and Woollf (2008) state that the highest and most persistent currents occur in the narrow gaps between islands. Indeed, Couch and Bryden (2006) recognise that hydrodynamic regimes are not necessarily mutually exclusive. As such, Black and Veatch Ltd. (2011) acknowledge that although hydraulic currents are the dominant feature, other mechanisms will contribute to the hydrodynamic characteristics of the Pentland Firth.

An alternative approach to characterising the hydrodynamic resource by Mackay (2007) and Mackay (2008) entails calculation of the total (kinetic plus potential) energy of a progressive tidal wave. In a progressive wave, peak tidal stream velocities occur at high and low water. Pugh (1987) distinguished this from hydraulic currents where peak velocities may occur at times other than high or low water. Black and Veatch Ltd. (2011) argue against this approach declaring that application to the Pentland Firth gives tidal stream velocities significantly lower than in reality. Salter and Taylor (2007) suggest that a phase difference of 46° exists between the tidal elevations and velocities—clearly not the behaviour of a progressive tidal wave. Despite these reservations, the principle underlying the analysis by Mackay (2008) is useful as it is the incoming oceanic tidal wave that is ultimately responsible for developing the pressure that induces the head difference across a hydraulic current. Davies et al. (2001), for instance, show that the elevation gradient caused by the eastwards sweep of the Atlantic Kelvin wave causes the large M2 tidal current velocities in the Pentland Firth.

If the phase relationship between the elevations and currents can be established then a more robust calculation of tidal energy budget can be made. Taylor (1919) equated the divergence of total energy flux across the boundaries of the Irish Sea with the energy dissipation from bed friction. A similar attempt to quantify the energy dissipation via bed friction in the Pentland Firth by Salter and Taylor (2007) and Salter (2009) suggested peak values of 53 GW and 104.8 GW, respectively. Whilst recognising the difference between peak and average dissipation, it is noteworthy that these values are far larger than several estimates of mean total power transferred by tides propagating onto the continental shelf west of Scotland.

The numerical model of Davies and Kwong (2000) and Davies et al. (2001) indicate a significant flux of energy from the deep water west of Scotland onto the shelf. From a coordinated set of offshore measurements Cartwright et al. (1980) calculate that the mean energy flux of the lunar semidiurnal tide onto the shelf between Northern Ireland and Norway is 60 GW. Based on a

![Fig. 1. Map of the North of Scotland and Orkney Islands showing location of the Pentland Firth and position of boundaries used in numerical model (dashed line).](image)
2. Tidal data sources for the Pentland Firth

As previously discussed, the availability of in situ data is a limiting factor for detailed studies of the Pentland Firth. Although a numerical model provides a useful alternative, model-data comparisons are a requirement to establish confidence in model results. Woolf and McIlvenny (2011) summarised the availability of tidal stream data sources for the Pentland Firth. In view of their sparseness of coverage, tidal diamonds were not considered a suitable data source for this study.

2.1. Tide gauge network

Tide gauges provide useful in situ measurements of sea surface elevation in harbour locations. The UK Strategic Tide Gauge...
Network constitutes 45 Class A tide gauges which are freely available from the British Oceanographic Data Centre (BODC). Only one gauge (Wick, 58.26N, 03.08W) was within the model domain (Fig. 2). Data collected by the tide gauge includes both astronomical and meteorological (wind and pressure) phenomena. The model, by contrast, only considered astronomical forcing. Meteorological and astronomical contributions were separated in the tide gauge signal by tidal harmonic analysis and removing the residual from measured values to derive predicted tidal elevation. The timing and magnitude of high and low water was extracted from the tidal signal at Wick. Using the simple interpolation procedure and constants detailed within UKHO (2009) the timing and magnitude of high and low water at four secondary ports around the Pentland Firth region were calculated; Scrabster, Gills Bay, Bur Wick and Muckle Skerry (Fig. 2).

2.2. Tidal stream measurement

Acoustic Doppler current profilers (ADCP) provide information on tidal currents throughout the water column. Three simultaneous 30 day moored ADCP deployments were conducted for the Navigation Safety Branch of the Maritime and Coastguard Agency. These devices were located through the centre of the Pentland Firth, separated by ca. 8 km (Fig. 2). The data, which offered 10-min averaged measurements at 4 m intervals through the water column (from 5 m above the seabed to ca. 10 m below the sea surface), were integral averaged over depth (1):

\[ \bar{U} = \frac{1}{D} \int U \, dz \]  

where, \( \bar{U} \), \( D \) and \( dz \) are, respectively, the depth-averaged velocity, water depth, and vertical bin size. High frequency variations were clearly detectable in the resulting depth-averaged ADCP time series. The source of these variations was unknown but may be an artefact of the mooring system. A low pass filter was thus employed to smooth the data and facilitate model-data comparison (Fig. 3). A cut-off frequency of 9.26 \times 10^{-4} \text{ Hz} (1/3 \text{ h}^{-1}) was applied (Legrand, 2009).

3. Numerical model of the Pentland Firth

3.1. Model overview

The MIKE21 Flow Model FM (Flexible Mesh) by DHI was used to construct a model of the Pentland Firth. The model benefits from a cell-centred finite-volume solution technique which enables an unstructured mesh to be applied around complex coastal areas. The flow model consists of a number of individual modules, of which only the hydrodynamic model was employed in the present investigation. The hydrodynamic model (henceforth HD model) is based on the two dimensional (depth-integrated) Reynolds’-averaged Navier–Stokes equations, applying the Boussinesq and hydrostatic approximations. In order to minimise computational solving time, the governing equations for both time integration and space discretisation were solved by a first-order scheme. A comprehensive description of the model can be found in DHI (2009b).

Only astronomical and Coriolis forcing were included in the model setup. It was assumed that density gradients were too low to contribute significantly. Wave radiation stress and direct meteorological forcing were not applied in the present study. Although episodic wind and wave forcing are significant in the region, their contribution was expected to be generally lower than the astronomical forcing. As there are no significant river discharges in the vicinity of the Pentland Firth, no additional flow sources were included.

3.2. Model domain and open boundaries

The model domain extended approximately 100 km west, east, and north of the Pentland Firth (Fig. 1). Five open-boundaries were specified by time varying water levels generated using the DHI global tidal database, KMS (Andersen, 1995; DHI, 2009a). The global model was based on the major diurnal and semi-diurnal tidal constituents (K1, O1, P1, Q1, M2, S2, N2, and K2), with a spatial resolution of 0.25 \times 0.25, which has been validated against TOPEX/POISEIDON altimetry data. Siting the open-boundaries far beyond the Pentland Firth was intended to reduce their influence on the solution (Venugapol et al., 2010).

An initial simulation was performed in order to verify that the model domain was sufficiently large to allow adequate propagation from the open boundaries. Initial evaluation involved comparison of observed and predicted tidal constituents and tidal ranges over a full spring-neap tidal cycle at one standard and four secondary ports in the region (Fig. 2).

3.3. Subgrid scale dissipation

The HD model uses the approach of Smagorinsky (1963) to dissipate energy at the subgrid-scale (SGS). The Smagorinsky model is based on the equilibrium hypothesis which implies that
the small scales dissipate entirely and instantaneously all the energy they receive from the large scale (Blazek, 2005). For incompressible flow the viscous stresses are related to the mean rate of momentum deformation by Newton’s law of viscosity (2):

$$\tau_{ij} = \nu S_{ij} = \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$  \hspace{1cm} (2)$$

where $S_{ij}$ is the mean deformation rate tensor and $\nu$ is the turbulent eddy viscosity, which is related to a characteristic length scale, $l$, based on the mesh spacing and a constant, $c_s$ (DHI, 2009b),

$$\nu = c_s l^2 \sqrt{2S_{ij}S_{ij}}$$  \hspace{1cm} (3)$$

from (3) it can be deduced that $c_s$, and consequently the shear stress removed at the SGS, $\tau_S$ (2), varies as the second power of the grid size. In other words, as the grid size reduces the dissipative and diffusive effect of the turbulence closure scheme reduces for a fixed $c_s$. The true geophysical dissipation is uncertain, but it is likely that excessive “numerical diffusion” is unrealistic. The most appropriate value for $c_s$ is not uniquely defined and in highly variable flow the sub-grid energy dissipation may be miscalculated (Jones et al., 2007). Setting $c_s$ to 0.28 (the recommended value (DHI, 2009b)) the mesh node separation within the Pentland Firth was varied in order to find a model setup that optimally calculated the bed resistance parameterised by the Manning Roughness, $M$ (m$^{1/3}$ s$^{-1}$) as

$$C_D = \frac{g}{(MD^{1/3})^2}$$  \hspace{1cm} (5)$$

where $g$, $D$ were the gravitational acceleration and local mean water depth (metres). To date, there have been no direct measurements of the bed roughness in the Pentland Firth. Salter (2009) provides an assortment of drag coefficients from the literature and suggests a value of 0.0086 for the Pentland Firth, based upon the depth-averaged value derived by Campbell et al. (1998). Cartwright et al. (1980), however, uses a lower drag coefficient of 0.0025 for shelf sea calculation. Similarly, Pugh (1987) recommends typical shelf sea values of 0.0015–0.0025. It will be shown in Section 4.1 that the optimal drag coefficient for the depth-averaged Pentland Firth model was 0.005 (double the value typical for shelf-sea tidal models).

Starting with the default MIKE21 bed resistance parameter ($M = 32$ m$^{1/3}$ s$^{-1}$, $C_D = 0.0022$ for $D = 80$ m), the HD model was run over a period of several days, coinciding with spring tides. Performance was assessed by comparing instantaneous current speeds against a limited set of in situ tidal stream measurements. Based on this result the value for the bed resistance was altered to iteratively achieve the best model-data correspondence. The value that minimised the error between the model and measured current speed was then assumed a priori to represent the optimal bed resistance throughout the entire Pentland Firth.

### 3.5. Calculation of tidal energetics

Mackay (2008) calculated the energy transmitted by a tide as the sum of the kinetic and potential energy Assuming the behaviour of a purely progressive wave. The group velocity and phase velocity of a shallow water wave (small water depth relative to wavelength) such as a tide propagating on the continental shelf are both determined by

$$c = \sqrt{gD}$$  \hspace{1cm} (6)$$

where $c$, $g$ and $D$ are the wave speed, gravitational acceleration and local mean water depth respectively. Tidal stream velocities in the progressive wave are related to the instantaneous sea level anomaly (or tidal elevation), $\zeta$, by

$$u_{\text{prog}} = \frac{c \zeta}{D}$$  \hspace{1cm} (7)$$

Thus, maximum current speeds occur at times of local high and low water ($\zeta_{\text{max/min}}$) and speed varies in phase with elevation. A progressive wave transfers energy in the direction of propagation (equivalent to the current direction at high water). The instantaneous energy flux (per unit width) associated with such a wave is determined by

$$E_{\text{prog}} = \rho g D \zeta c$$  \hspace{1cm} (8)$$

In coastal waters, however, adherence to progressive wave behaviour is unlikely and a more universal means with which to calculate the energy flux should be applied. Taylor (1919) equates the total energy flux across a boundary to the sum of the rate of work done against the pressure head and the fluxes of potential energy and kinetic energy. The work done against direct gravitational forcing was also accounted for and was found to provide only a small contribution to the total flux. Garrett (1975) provides a more precise formulation for the mean energy balance that accounts for an additional corrective term relating to the equilibrium tide. Nevertheless, the energy flux of the tidal wave and the dissipation due to bed friction are usually the largest terms in the energy balance (Howarth and Pugh, 1983). It is expected here, therefore, that the energy balance in the Pentland Firth is dominated by the horizontal energy flux and the energy dissipation via bed friction and associated turbulent processes.

Three transects were chosen to represent the geographic boundaries of the Pentland Firth (Fig. 2). The Western boundary (B1) spanned 14 km from Dunnet Head (Mainland Scotland) to Torr Ness (Hoy, Orkney). The Northern boundary (B2) crossed the 7 km gap that marks the entrance to Scapa Flow, between the Orkney Islands of South Walls and South Ronaldsay. The Eastern boundary (B3) from Duncansby Head to Brough Ness (South Ronaldsay) was 11 km in length. Model surface elevation ($\zeta$), mean water depth ($D$), and depth-averaged tidal current velocities ($\overline{U}$ = $\overline{U}$, $\overline{V}$) were extracted at 500 m intervals across each boundary. In reality, tidal currents vary in three-dimensions; however Davies and Kwong (2000) suggest that depth mean values ($\overline{U}$) are more appropriate for energy flux calculations. The instantaneous energy flux (per unit width) was then calculated for each time step using

$$E = \rho g (\overline{U}^2 + \overline{V}^2) \overline{U} + 1/2 \rho (D + \zeta) \overline{U}^3$$  \hspace{1cm} (9)$$

The first term in Eq. (9) includes contributions from both the potential energy transferred and work done against pressure, whereas the second term represents kinetic energy flux. As $\zeta = D$, and assuming the relationship in (7), higher powers of $\zeta$ and $\overline{U}$ are often omitted, reducing (9) to (8). This, however, invokes an underlying assumption of conformance to a free progressive
wave. All terms in Eq. (9) were retained so as not to bias results by that assumption. The mean energy flux over a tidal cycle across each boundary was then calculated from (10):

$$\overline{E_B} = \int_B \langle \rho \mathbf{U} \mathbf{U} \rangle \, dB$$

(10)

where the $\langle \rangle$ brackets denote the mean value and $\mathbf{n}$ is the inwardly pointing unit vector normal to an element $dB$ on the boundary $B$. The three defined boundaries enclose the “greater Pentland Firth”. If the fluxes across the boundaries are totalled then a positive sum implies an energy flux of that magnitude into the greater Pentland Firth. It is proposed that this net energy flux, for the most part, must be balanced by energy dissipation due to bed friction, $E_c$ within the Pentland Firth. From (4) the mean value was found as

$$\overline{E_c} = \int_A \rho C_D U^3 \, dA$$

(11)

where the drag coefficient $C_D$ was defined via (5) within each mesh element $dA$, within $A$, the area enclosed by the boundaries $B_1$, $B_2$, and $B_3$ (Fig. 2).

4. Results

4.1. Model optimisation

4.1.1. Tidal elevations

The initial model experiment was intended to verify that the model domain was sufficiently large to allow adequate propagation from the open boundary. The default model bed resistance value (32 m$^{1/3}$ s$^{-1}$, $C_D=0.0022$ for $D=80$ m) was used. Predicted and observed tidal elevations were compared for a full spring-neap tidal cycle (Table 1). Spring and neap tidal ranges were compared as was the timing of high water. Assessment was based on performance criteria as specified in UKFWR (1993):

- Tidal elevations: RMS(error) $< 10\%$ on spring tide and 20\% on neap tide ranges; and
- Timing of high water RMS error $< 25$ min.

Comparison between the predicted and observed tidal elevations at Wick (Standard Port) showed RMS spring range error (8.0\%), RMS neap range error (9.59\%), and an error in timing of high water (-6 min) within the bounds of the prescribed model performance criteria (Table 1). The amplitude and phase of two semiidiurnal (M2, S2) and two diurnal (K1, O1) tidal constituents are also provided by UKHO (2009), and are compared with the constituents extracted from model results (Table 2). For Wick, the model overestimated the amplitude of the principal semiidiurnal M2 constituent by 0.09 m ($-9\%$). Of greater significance is that the model accurately predicted the phasing of the tide at the ports flanking the Pentland Firth (Wick and Scrabster). It is proposed that this relationship is most important for replicating the flow through the Pentland Firth.

The timing and magnitudes of high and low water calculated through interpolation and extrapolation of the Standard Port data are compared with model predictions at four secondary ports around the Pentland Firth (Table 1). For the secondary ports, the model results agreed sufficiently with the interpolated results for three of these ports based on the chosen performance criteria, namely: Scrabster, Muckle Skerry, and Gills Bay. The one port that did not satisfy all the performance criteria (Bur Wick) failed with regards to only one measure, the timing of high water. In general, the model underestimated the amplitude of the diurnal tidal constituents and overestimated the amplitude of the semiidiurnal S2 constituent (Table 2). The amplitude and phase of the principal semiidiurnal M2 tide, however, showed overall good agreement with the published data.

### Table 2

<table>
<thead>
<tr>
<th>M2</th>
<th>S2</th>
<th>K1</th>
<th>O1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pha</td>
<td>Amp</td>
<td>Pha</td>
<td>Amp</td>
</tr>
<tr>
<td>Charted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wick</td>
<td>322</td>
<td>0.02</td>
<td>000</td>
</tr>
<tr>
<td>Bur Wick</td>
<td>287</td>
<td>0.88</td>
<td>322</td>
</tr>
<tr>
<td>Gills Bay</td>
<td>268</td>
<td>1.12</td>
<td>300</td>
</tr>
<tr>
<td>Scrabster</td>
<td>247</td>
<td>1.35</td>
<td>280</td>
</tr>
</tbody>
</table>

| Model ($C_D=0.0022$) |          |          |          |
| Wick     | 322      | 1.11     | 357      | 0.49     | 178      | 0.07     | 026      | 0.12     |
| Bur Wick | 302      | 0.77     | 336      | 0.35     | 169      | 0.09     | 014      | 0.12     |
| Gills Bay| 269      | 1.06     | 300      | 0.48     | 151      | 0.07     | 014      | 0.09     |
| Scrabster| 243      | 1.33     | 275      | 0.64     | 134      | 0.09     | 035      | 0.09     |

| Model ($C_D=0.0005$) |          |          |          |
| Wick     | 322      | 1.08     | 357      | 0.48     | 179      | 0.07     | 029      | 0.11     |
| Bur Wick | 297      | 0.80     | 329      | 0.36     | 164      | 0.08     | 018      | 0.10     |
| Gills Bay| 266      | 1.08     | 297      | 0.50     | 147      | 0.08     | 010      | 0.10     |
| Scrabster| 241      | 1.30     | 273      | 0.63     | 133      | 0.09     | 035      | 0.09     |

### Table 3

RMS (%) magnitude and peak timing difference between values of model and measured current speed for different bed roughnesses. $C_D$ calculated for water depths of 80 m ($5$).

<table>
<thead>
<tr>
<th>Friction</th>
<th>%RMS current speed error</th>
<th>At peak current (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (m$^{1/3}$ s$^{-1}$)</td>
<td>Friction, $C_D$</td>
<td>ADCP1</td>
</tr>
<tr>
<td>32.0</td>
<td>0.0022</td>
<td>47.4</td>
</tr>
<tr>
<td>30.0</td>
<td>0.0025</td>
<td>41.0</td>
</tr>
<tr>
<td>25.0</td>
<td>0.0036</td>
<td>24.8</td>
</tr>
<tr>
<td>22.5</td>
<td>0.0045</td>
<td>14.2</td>
</tr>
<tr>
<td>21.3</td>
<td>0.0050</td>
<td>9.96</td>
</tr>
<tr>
<td>19.5</td>
<td>0.0060</td>
<td>10.8</td>
</tr>
</tbody>
</table>

### Table 1

Errors in timing of high water and tidal range for standard and secondary ports around the Pentland Firth.

<table>
<thead>
<tr>
<th>Friction, $C_D$</th>
<th>Standard Port</th>
<th>Secondary Port</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wick</td>
<td></td>
<td>Scrabster</td>
</tr>
<tr>
<td>Wick</td>
<td></td>
<td>Muckle Skerry</td>
</tr>
<tr>
<td>Wick</td>
<td></td>
<td>Bur Wick</td>
</tr>
<tr>
<td>Wick</td>
<td></td>
<td>Gills Bay</td>
</tr>
<tr>
<td>Spring tidal range RMS error (%)</td>
<td>0.0022</td>
<td>8.00</td>
</tr>
<tr>
<td>Neap tidal range RMS error (%)</td>
<td>0.0022</td>
<td>13.5</td>
</tr>
<tr>
<td>Timing error of High water, $\Delta t$</td>
<td>0.0022</td>
<td>-6</td>
</tr>
</tbody>
</table>

Comparison between the predicted and observed tidal elevations at Wick (Standard Port) showed RMS spring range error (8.0\%), RMS neap range error (9.59\%), and an error in timing of high water (-6 min) within the bounds of the prescribed model performance criteria (Table 1). The amplitude and phase of two semiidiurnal (M2, S2) and two diurnal (K1, O1) tidal constituents are also provided by UKHO (2009), and are compared with the constituents extracted from model results (Table 2). For Wick, the model overestimated the amplitude of the principal semiidiurnal M2 constituent by 0.09 m ($-9\%$). Of greater significance is that the model accurately predicted the phasing of the tide at the ports flanking the Pentland Firth (Wick and Scrabster). It is proposed that this relationship is most important for replicating the flow through the Pentland Firth.
We must consider the interpolated data used to assess the model results. UKHO (2009) state that the quality of data at the secondary ports, upon which the corrections are based, are variable. Important local geographic features should also be identified. Bur Wick, for example, is shallow, may experience frequent wetting and drying, and is located adjacent to a headland tidal race (known locally as the Liddel Eddy). These factors may influence both the model and measured tidal signal at Bur Wick.

The model reproduced the majority of the tidal signal at the standard port (Wick). The amplitudes and phases of the principal tidal constituents showed overall good agreement with published data. The performance of the model in reproducing the tidal signal in and around the waters of the Pentland Firth was regarded as satisfactory and the domain and boundaries were therefore retained for the subsequent model experiments.

4.1.2. Bed resistance

The HD model was run over a period of several days at spring tides using a range of bed resistance values. Model current speeds were compared against depth-averaged velocities from the three ADCP stations and performance was assessed based on criteria specified in UKFWR (1993):

- Current speed RMS(error) from < 10% to 15%; and
- Timing error of maximal current speed < 25 min.

Using the default bed resistance value (32 m$^{1/3}$ s$^{-1}$, $C_D=0.0022$ for $D=80$ m) results fall outside of the chosen performance criteria, with RMS current speed errors of up to 47.4% (Table 3). Increasing the bed friction results in a decrease of current speed error at the three locations assessed, accompanied by a decrease in the timing error of peak currents. No single bed resistance value minimised the errors at the three locations simultaneously; however, all sites satisfied the performance criteria for values of 22.5 and 21.3 m$^{1/3}$ s$^{-1}$. Although the resulting drag coefficients were larger than typical values for shelf seas ($C_D\approx0.0050$ for $D=80$ m), this model setup replicated the measured current speed distribution at each ADCP station with overall good accuracy (Fig. 4).

Changing the bed resistance during the calibration of model tidal currents necessarily impacted on the predicted tidal current speed distributions over 24-h, spring tide, at ADCP1 (top), ADCP2 (middle), ADCP3 (bottom). See Fig. 2 for geographical locations. Model results are shown every 10th time step for clarity.
elevations (Section 4.1.1). A check of the predicted tidal elevations (Table 1) and tidal constituents (Table 2), however, showed that results remain valid, and in most cases improved, with the larger mode bed resistance.

4.2. Tidal energetics

Using the optimised model setup, with uniform drag coefficient $C_D=0.005$, the HD model was used to simulate the flow in the Pentland Firth over one lunar month, equivalent of two full spring-neap tidal cycles (29.6 days, plus an additional 48 h for initialisation). The model time step was 60 s. Surface elevation and current velocities were selected as model outputs which were saved at every time step.

Model surface elevations for two consecutive semi-diurnal tidal cycles were used to calculate the current speeds in a progressive tidal wave using (7). Model current speeds were extracted over the same period and mean values were normalised against the progressive wave current speed. This revealed model current speeds that exceeded progressive wave current speeds within, and beyond, the geographic boundaries of the Pentland Firth (Fig. 5). This phenomenon was detectable up to ca. 3.75 W, approximately 20 km west of boundary B1, and a similar distance beyond the eastern boundary B2 (Fig. 2). Mean current speeds within the channel were typically four times the equivalent progressive wave current speed, and as much as 10-times through narrow gaps between islands.

Model tidal stream velocities and elevations were extracted at 500 m intervals along the three boundaries defining the geographic extent of the Pentland Firth (Fig. 2). The tidal energy flux across each boundary was calculated from (10). The result showed a mean tidal energy flux across the western boundary (B1) of 8.97 GW, from west-to-east (Table 4). A subsequent harmonic analysis suggested that the semi-diurnal M2 tide constitutes 80% (7.25 GW) of this flux. The mean energy flux across the eastern boundary (B3) was also from west-to-east, with magnitude 3.27 GW. The northern boundary (B2) contributed relatively little to the overall energy budget, the net flux being two orders of magnitude lower than across either B1 or B3.

To gain better insight into the change of the mean tidal energy flux the above calculations were repeated for a further 21 cross-sections along the length of the Pentland Firth (from 3.34 W to 2.95 W). Sections were separated by 1 km relative to the position of their midpoints and aligned normal to the mean flood/ebb flow direction. Decreasing tidal energy flux was observed from west-to-east; however, the rate of decrease was not uniform and generally increased eastwards (Fig. 6). The maximal decrease in the tidal energy occurred between 3.15 W and 3.05 W. Tidal analysis of elevation at the midpoint of each section also revealed decreasing amplitude and increasing phase of the M2 tidal constituent from west-to-east (Fig. 7). The most rapid change in M2 tide again occurred between 3.15 W and 3.05 W. It is apparent (for example, by referring to Fig. 2) that the most rapid changes in energy flux, tidal phase, and tidal amplitude coincide with the constrictions in flow associated with the islands of Stroma and Swona.

The energy dissipated via bed friction within the area bounded by B1, B2 and B3 was calculated using model current speeds and drag coefficient using the quadratic friction law in (11). The mean rate of frictional energy dissipation was 5.24 GW when averaged over two spring-neap tidal cycles (Table 4). The majority of this dissipation occurred east of 3.2 W, through constrictions, and around headlands (Fig. 8). The most intense areas of dissipation were found through the Outer Sound (between the islands of Stroma and Swona), the Inner Sound (between Stroma and Scottish Mainland), and off the promontory of South Ronaldsay.

As stated in Section 4.2, the drag coefficient for the optimised model was larger than typical shelf seas values. To test the significance of this result on the tidal energy budget, we performed an additional model using lower drag ($C_D=0.0025$). In this setup, model current speeds exceeded the measured values (Fig. 4). Although there was an increase in the mean tidal energy across both the western and eastern boundaries, the net flux into the Pentland Firth was comparable to the higher friction case (Table 4). Similarly, the energy dissipation via bed friction showed only small change from the previous model.

<table>
<thead>
<tr>
<th>Friction, $C_D$</th>
<th>Boundary Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>West (B1)</td>
</tr>
<tr>
<td>Total energy flux, $\Delta E$ (GW)</td>
<td>0.0050</td>
</tr>
<tr>
<td>Kinetic energy flux, $\Delta E_K$ (MW)</td>
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</tr>
<tr>
<td>Frictional dissipation, $\Delta E_f$ (GW)</td>
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</tr>
<tr>
<td>$\Delta E_f$ (GW)</td>
<td>0.0025</td>
</tr>
</tbody>
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Fig. 6. Mean tidal energy flux along the Pentland Firth against longitude (W) of cross-section at midpoint.

Fig. 7. Amplitude (■) and phase (□) of M2 tidal elevations and phase of M2 tidal currents (○) along the Pentland Firth, against longitude (W) of cross-section at midpoint.
5. Discussion

The Pentland Firth experiences tidal stream velocities that greatly exceed the current speed of a freely progressive tidal wave subject to the same free surface elevations and water depth. Results from a two dimensional hydrodynamic model show that from a position 20 km beyond the geographic entrance to the strait, mean tidal streams are accelerated up to 10-times the equivalent free wave current speed (Fig. 5). The tidal conditions at locations flanking the Pentland Firth are not, by themselves, remarkable by shelf sea standards (Table 2). The amplitude of the semidiurnal M2 tide at the ports of Scrabster (1.30 m) and Wick (1.08 m) are both moderate and of similar magnitude. On the other hand, the scales of both the Atlantic and North Sea tide are far greater than the width of the Pentland Firth. The consequence of this geographical quirk is a shift of both tidal phases and amplitudes over a relatively short distance. For example, model results show an M2 phase difference of 50° (ca. 1.7 h) and a decrease in M2 tidal amplitude of approximately 0.31 m over just 23 km (Fig. 7). Thus, peculiarities of geography act to create exceptional currents, where the regional tides are in other respects quite ordinary. In this way, the Pentland Firth flow can be described, in one sense, as a hydraulic current, but as explained below that current is also fuelled by the funnelling of the shelf tides.

The North Atlantic tide provides energy through the western boundary of the Pentland Firth at a mean rate of 8.97 GW. Tidal analysis indicates that 80% (7.25 GW) of this originates from the M2 tidal constituent. The relationship between the phases of M2 elevations and currents at the western boundary approaches that of a progressive tidal wave; currents lead the elevations by 4° (<10 min), on average (Fig. 7). While the phase of the M2 tidal elevation increases from west-to-east, the phase of the M2 tidal currents show only slight overall increase, and even decreases between 3.2 W and 3.05 W. The overall effect is an increase in the phase lag eastwards producing behaviour more closely resembling that of a standing wave, and associated with this, a reduction in the tidal energy flux; the mean rate of energy exiting across the eastern boundary is 3.27 GW. Including the comparatively small contributions from the Northern boundary, the remainder of 5.62 GW is lost within the Pentland Firth (Table 4 and Fig. 6). The peculiarities of geography are such that very little of the energy approaching from the west is reflected or diverted away from the Pentland Firth; instead approximately 60% is lost within the locale, while most of the remainder is transmitted to the North Sea.

Model results reveal that the rate at which tidal energy is lost through the Pentland Firth is not constant and shows a spatial dependence that corresponds with its internal geography. From 3.4 W to 3.2 W there is a measured decrease in tidal energy flux (Fig. 6) and M2 tidal amplitude (Fig. 7). Here the channel is relatively wide and uniform and the hydraulic current dominates. Between 3.2 W and 3.05 W, however, islands complicate this geography and create narrower branched channels. The flow accelerates through the constriction in response to the maintenance of continuity (so called tidal streaming) with more rapid conversion of potential to kinetic energy. This conversion is manifest by the large drop in tidal amplitude (Fig. 7) and increased tidal stream velocities (Fig. 5). Additionally, as frictional dissipation scales with the cube of the tidal stream velocities, the rate of tidal energy loss increases (Fig. 6). The mean net export of kinetic energy across the boundaries of the Pentland Firth is relatively small representing less than 10% of the total mean energy flux exiting the system (Table 4).

The energy dissipated via bed friction is calculated using the quadratic friction law. The results show a mean rate of energy dissipation of 5.24 GW over two consecutive spring-neap tidal cycles. This figure agrees well with the 5.62 GW net energy flux calculated across the boundaries of the Pentland Firth (Table 4). The mean dissipation is a useful guide to the absolute upper limit of tidal energy extraction (Arbic and Garrett, 2010). The majority of this dissipation occurs east of 3.2 W, through constrictions and around headlands (Fig. 8), in particular; through the Outer Sound (between the Islands of Stroma and Swona); the Inner Sound (between Stroma and mainland Scotland); and off the promontory of South Ronaldsay. This result shows good agreement with the above geographical interpretations of the total energy flux.

The energy budget of the Pentland Firth can be summarised by inference from the model experiments in the following way: the net source of energy supply, tidal forcing from the North Atlantic Ocean, goes to export of tidal energy out of the Pentland Firth into the North Sea and dissipation via bed friction within the Pentland.
Firth. It is difficult to separate the contributions due to the two dominant hydrodynamic regimes, the regional scale hydraulic current and localised tidal streaming. The major advantage of employing such generic hydrodynamic characterisations to tidal energy sites (e.g. Couch and Bryden, 2006; Black and Veatch Ltd., 2011) is the savings offered in terms of time and costs compared to conducting a number of individual case-by-case assessments. In the context of characterising tidal stream resource on a large (e.g. national) scale such an approach is instructive. It is evident, however, that the complexities of the real tidal environment require detailed, site-specific models to capture all of the involved phenomena.

Model tests were performed to achieve an optimal model setup and ensure the suitability of the domain, boundary conditions, and mesh element size. The “fixed” offshore boundaries, generated using a global tidal model, were shown to be suitable for this investigation of the Pentland Firth. We note, however, that if our model was elaborated to include extensive tidal energy extraction these boundaries may cease to be suitable as “artificial” reflected waves cannot propagate across the fixed boundaries. Garrett and Greenberg (1977) explored this “open boundary problem” in detail. Adcock et al. (2011) discuss various mitigation strategies for the open boundary problem that modellers of tidal energy extraction may consider. The model drag coefficient was determined via an iterative calibration against tidal stream velocity measurements from three acoustic current meters. The optimal value is double the recommended value for a typical shelf sea. As highlighted by French (2010), the rather subjective nature of the model calibration performed invokes restrictive assumptions as regard to the nature of model errors. Our results should therefore be treated with a degree of caution and the potential for errors in the data used to calibrate the model should be recognised. We identify the following three issues. Firstly, high frequency variations detected in the current speed observations was considerable such that, to facilitate calibration, a filter was applied to smooth the signal data. Analysis of the filtered time series, however, show rather flat topped peaks and hence suggests constrained maximum current speeds (Fig. 3). Secondly, the spatial density of calibration data is small relative to the size of the model domain: three measurement stations are located through the centre of the channel and separated by ca. 8 km (Fig. 2). As this data represents only a subset of the model domain, the optimal bed resistance is assumed a priori to apply uniformly. Whereas a constant seabed drag coefficient may not be fully realistic, paucity of information means that it is difficult to reflect physical differences in the seabed with any great certainty. Characterising the seabed structure of the Pentland Firth is an ongoing area of research. For example, photographic evidence reveals a heterogeneous seabed containing rocky boulders (Moore, 2009). The presence of boulders may partly explain the large bed resistance values as form drag from large obstacles will manifest as pseudo bed shear stress in the two dimensional shallow-water wave before the actual Pentland Firth due to the high gradients in elevation. At the western end of the Pentland Firth and the North Atlantic elevation and current are almost in perfect phase, similar in that respect to an easterly propagating wave, but further east, while the phase of the currents barely changes, the phase of elevation lags further and the amplitude of the tidal components also reduces. A reduction in energy flux is associated with these amplitude and phase changes, such that the majority (≈60%) of the incoming energy is lost within the Pentland Firth.

The flow within the Firth itself is largely hydraulic in nature with potential energy converted to kinetic energy, most of which is then rapidly dissipated by bed friction. The majority of the conversion to kinetic energy and hence to frictional dissipation occurs near the constrictions associated with the small islands of Stroma and Swona and the nearby headlands of mainland Scotland and Orkney. Thus “tidal streaming” plays a significant role in the kinetics of the flow but the dynamics is primarily hydraulic in nature.

Real tidal sites rarely conform to a simple paradigm of flow, though these paradigms can be useful to understanding. By constructing a reasonable model of a major tidal channel, we are in a better position to consider the scope for exploiting that site (using turbines extracting hydro-kinetic energy or otherwise). It is clear that any exploitation is unlikely to approach the flux of energy of up to 10 GW from the North Atlantic tide into the Pentland Firth.
and that reflection or diversion of this energy needs to be avoided. Further, it is clear that both hydraulics and tidal streaming are essential to producing the exceptionally high currents near Stroma and Swona and it is these phenomena that make a significant fraction of the total energy flux available as hydrokinetic energy. It is beyond the scope of this work to consider the practically exploitable resource further, but it is clear that a model that correctly simulates the basic dynamics will be an essential tool for such investigations.

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(last accessed March.12).


