6.3 Velocity distribution, bed roughness and friction factors in unvegetated channels

6.3.1 Viscous sublayer

On the bottom of an unvegetated channel there is no turbulence and the turbulent shear stress \( \tau_t = 0 \), and therefore in a very thin layer above the bottom, viscous stress is dominant, and hence flow is laminar in that layer called viscous sublayer.

Above the viscous sublayer the turbulent stress dominates and the total stress can be calculated from Eq. (6-16), i.e. as the sum of viscous and turbulent shear stresses.

\[
\tau_z = \tau_v + \tau_t = \rho g (h - z) S_0 \tag{6-16}
\]

The total stress decreases linearly towards zero when approaching the water surface (\( z \) approaches water depth \( h \)). The distribution of the shear stress as a function of \( z \) is completely analogous with the hydrostatic pressure \( p = \rho g (h - z) \) with the exception of the viscous sublayer close to the bottom.

6.3.2 Scientific classification of flow layers in unvegetated channels

According to Liu (2001) the so called scientific classification of flow layers is the one shown in Fig. 6-4.
Fig. 6-4. Scientific classification of flow regions in unvegetated channels (not to scale). Adapted from Liu (2001).

Near the bottom there is the thin viscous sublayer where there is almost no turbulence. Measurements show that the viscous shear stress in this layer is constant and equal to the bottom shear stress $\tau_b$. Flow in this layer is laminar. In the transition layer viscosity and turbulence are equally important. In the turbulent logarithmic layer viscous shear stress is negligible and the turbulent shear stress is equal to the bottom shear stress. The Prandtl's mixing length theory was developed for this layer and it leads to the logarithmic velocity profile as shown later on. The turbulent outer region consists of about 80-90% of the total region and velocity is relatively constant due to the strong mixing of the flow (Liu 2001).

### 6.3.3 Bed roughness $k_s$ in unvegetated channels

Bed roughness $k_s$ is very often also called roughness height (karkeuskorkeus in Finnish) or equivalent roughness (ekvivalentti karkeus).

One important question is how to determine the bed roughness $k_s$. Nikuradse made his experiments by gluing grains of uniform size to pipe surfaces. In completely flat bed consisted of uniform spheres the bed roughness $k_s$ would be the diameter of the grains.
This cannot be found in nature, where the bed material is composed of grains with different size and bottom itself is not flat but it includes ripples or dunes (see Fig. 6-5).

According to Liu (2001), the following $k_s$ values have been suggested based on different type of experiments:

- concrete bottom $k_s = 0.001 - 0.01$ m
- flat sand bed $k_s = (1 - 10)d_{50}$
- bed with sand ripples $k_s = (0.5 - 1.0)H_r$ : $H_r =$ ripple height (m)

Fig. 6-5. Ripples and dunes (adapted from Liu 2001).

### 6.3.4 Characterisation of smooth and rough flow in unvegetated channel

It is necessary to characterise the flow as hydraulically smooth or rough since it influences e.g. the thickness of the viscous sublayer etc. A very big series of experiments were carried out by Nikuradse for pipe flows. He introduced the concept of equivalent grain roughness $k_s$, which is usually called bed roughness for open channel flow. Based on the experiments it was found that the following criteria can be used to characterise if flow is hydraulically smooth, rough or in the transitional zone:
\[
\frac{\nu_* k_s}{v} \leq 5; \quad \text{hydraulically smooth flow}
\]
\[
\frac{\nu_* k_s}{v} \geq 70; \quad \text{hydraulically rough flow}
\]
\[
5 \leq \frac{\nu_* k_s}{v} \leq 70; \quad \text{hydraulically transitional flow}
\] (6-18)

where \(\nu_*\) is the friction velocity calculated using Eq. (6-15) and \(v\) is the kinematic viscosity.

### 6.3.5 Logarithmic velocity distribution in turbulent layer

It was described in section 6.3.2 that the measurements show that the turbulent shear stress is constant in the turbulent logarithmic layer and it equals the bottom shear stress. By assuming that the mixing length is proportional to the distance to the bottom, \(l_m = \kappa z\) (\(\kappa\) is von Karman constant), Prandtl obtained the logarithmic velocity profile. By the modifications done to the original theory, the logarithmic velocity profile applies also in the transitional layer and in the turbulent outer layer. Measured and computed velocities show reasonable agreement. This means that from an engineering point of view, two different velocity profiles need to be considered (Liu 2001):

- logarithmic velocity distribution, which covers the transition layer, turbulent logarithmic layer and turbulent outer layer from Fig. 6-4.
- velocity profile in the viscous sublayer

In the turbulent layer the total shear stress is assumed to contain only the turbulent shear stress. The total shear stress increases linearly with depth

\[
\tau_t(z) = \tau_b \left(1 - \frac{z}{h}\right)
\] (6-19)

According to Prandtl's mixing length theory shown in section 6.2.2 and Eq. (6-6)

\[
\tau_t = \rho l_m^2 \left(\frac{dv}{dz}\right)^2
\] (6-6)
Now comes out the modification of the original theory. Instead of assuming that the mixing length \( l_m = \kappa z \) like Prandtl assumed, \( l_m \) is replaced by equation

\[
l_m = \kappa z (1 - \frac{Z}{h})^{0.5}
\]  

(6-20)

It is possible to combine (6-20) and (6-6) to get

\[
\frac{dv}{dz} = \frac{\sqrt{\tau_b / \rho}}{\kappa z} = \frac{v_*}{\kappa z}
\]  

(6-21)

Eq. (6-21) can be integrated from \( z_0 \) to \( h \) to get the logarithmic velocity profile.

\[
v(z) = \frac{v_*}{\kappa} \ln \left( \frac{Z}{z_0} \right)
\]  

(6-22a)

The integration constant \( z_0 \) is the elevation corresponding to zero velocity (see Fig. 6-6).

Note: Most often the "universal" logarithmic velocity distribution is shown in the form:

\[
\frac{v(z)}{v_*} = \frac{1}{\kappa} \ln \left( \frac{Z}{k_s} \right) + 8.5
\]  

(6-22b)

See next page for comparison of (6-22a) and (6-22b)!

\textit{Note the similarity to the calculation of aerodynamic resistance \( r_a \) needed in the estimation of potential evapotranspiration rate using the Penman-Monteith equation! In these calculations the roughness length is usually around 10-13 % of the crop height!}
Fig. 6-6. Velocity distribution in hydraulically smooth and rough flow (graphs not to scale).

The integration constant $z_0$ (m) is here based on the study conducted by Nikuradse for pipe flows.

$$z_0 = \begin{cases} 
0.11 \frac{V}{V^*} & \frac{V^*k_s}{V} \leq 5; \text{ hydraulically smooth flow} \\
0.033k_s & \frac{V^*k_s}{V} \geq 70; \text{ hydraulically rough flow} \\
0.11 \frac{V}{V^*} + 0.033k_s & 5 \leq \frac{V^*k_s}{V} \leq 70; \text{ hydraulically transitional flow}
\end{cases} \quad (6-23)$$

NOTE! Eqs. (6-22a) and (6-22b) are exactly the same since (6-22b) is valid from $z$-values which give $v(z)$ greater than or equal to zero. $v(z)$ given by (6-22b) is zero when

$$z(v = 0) = k_s e^{-8.5\kappa} = 0.033k_s = z_0 \quad \text{(for rough flow)} \quad (6-22c)$$

The friction velocity $v^*$ is the fluid velocity very close to the bottom and it is the flow velocity at elevation $z = z_0 e^{\kappa}$, i.e.

$$v = z_0 e^{\kappa} = v^* \quad (6-24)$$
6.3.6 Velocity profile in the viscous sublayer

In the case of hydraulically smooth flow there is a viscous sublayer whereas in the rough flow there is a height $z_0$ with zero flow (Figs. 6-6 and 6-7). Viscous shear stress is constant in the viscous sublayer and it is equal to the bottom shear stress as shown in Fig. 6-4.

$$\tau_v = \rho \nu \frac{dv}{dz} = \tau_b$$  \hspace{1cm} (6-25)

Eq. (6-25) can be integrated assuming that velocity $v=0$ when $z=0$. Integration gives

$$v(z) = \frac{\tau_b}{\rho} z = \frac{v^2_*}{\nu} z$$  \hspace{1cm} (6-26)

giving a linear velocity distribution in the viscous sublayer. The thickness of the viscous sublayer can be obtained by finding the $z$ value where the logarithmic velocity distribution intersects the linear distribution giving a theoretical thickness of the viscous sublayer $\delta_v$ (Liu 2001).

$$\delta_v = 11.6 \frac{\nu}{v_*}$$  \hspace{1cm} (6-27)

Summary of the velocity profiles in the hydraulically smooth and rough flow is given in Fig. 6-7.
Fig. 6-7. Summary of the velocity profiles in hydraulically smooth and hydraulically rough flow (in the graph $u(z)$ refers to $v(z)$ and $u_*$ refers $v_*$).

### 6.3.7 Drag force

Flowing water moving past an object will exert a force called drag force

$$F_D = \frac{1}{2} \rho C_D A v^2 \quad (6-28)$$

where $C_D$ is the coefficient and $A$ is the projected area of the object to the flow direction. Values of $C_D$ are usually around 1.0...2.0 but experiments are needed for more accurate determination of the values. Drag force comes partly from the skin friction when water moves around the object and partly from the pressure the moving water exerts on the object.

Correspondingly to the drag force another force called lift force $F_L$ can be defined. It has the same shape than the drag force:

$$F_L = \frac{1}{2} \rho C_L A v^2 \quad (6-29)$$

where $C_L$ is the lift coefficient which also need to be determined experimentally.
6.3.8 Chézy, Manning and Darcy-Weissbach friction coefficients

The three most often used equation for calculating friction losses $h_f$ (Eqs. 6-30) are the ones known as Chézy, Manning and Darcy-Weissbach equations.

$$h_f = \frac{1}{C^2} \frac{L v^2}{R} \quad \text{Chézy}$$

$$h_f = n^2 \frac{L v^2}{R^{4/3}} \quad \text{Manning} \quad (6-30)$$

$$h_f = f_{DW} \frac{L}{4R} \frac{v^2}{2g} \quad \text{Darcy – Weissbach}$$

where $L$ is the length of the river section and $R$ is hydraulic radius. $C$ is the Chézy coefficient, $n = 1/k_{St}$ is the Manning coefficient (the inverse of $n$ is $k_{St}$, called the Strickler coefficient) and $f_{WD}$ is the Darcy-Weissbach friction coefficient originally developed for pipe flow. Note the $h_f$ is linearly related to $f_{WD}$ but to $C^2$ and $n^2$.

There is the following connection between the different friction coefficients:

$$f_{DW} = 8g R^{-1/3} n^2 = 8g C^{-2} \quad (6-31)$$

The three different friction factors are related to calculation of average velocity $v$ (m s$^{-1}$) in the river as follows:

$$v = C \sqrt{RS_f} \quad \text{Chézy}$$

$$v = \frac{1}{n} R^{2/3} \sqrt{S_f} \quad \text{Manning} \quad (6-32)$$

$$v = \frac{8g}{f_{DW}} \sqrt{RS_f} \quad \text{Darcy – Weissbach}$$

where $S_f$ is the energy slope (friction slope) and in uniform flow $S_f$ can be replaced by the bottom slope $S_o$. 

The next step is to provide relationships between bed roughness $k_s$ and different friction factors in unvegetated channels.

**Chézy coefficient versus bed roughness $k_s$**

Liu (2001) has derived the following relationships for Chezy coefficient $C$ and bed roughness $k_s$ in a broad channel (bottom width $B_B >> h$, i.e. $R$ can be replaced by $h$). If one compares the well-known Chézy equation

$$v = C\sqrt{RS_f} \quad (6-31)$$

and the equation for friction velocity $v*$, (note $S_0$ replaced by $S_f$)

$$v* = \sqrt{gRS_f} \quad (6-15)$$

it is possible to derive a connection between $C$, $v*$ and average velocity $v$.

$$C = \frac{v}{v*} \sqrt{g} \quad (6-33)$$

In the next step it is necessary to calculate the average velocity from the logarithmic velocity profile by integrating it from $z_0$ to $h$ (Liu 2001).

$$v = \frac{1}{h} \int_{z_0}^{h} v(z)dz = \frac{v*}{\kappa h} \int_{z_0}^{h} \ln(z/z_0)dz = \frac{v*}{\kappa} \ln\left(\frac{h}{z_0} - 1 + \frac{z_0}{h}\right) \approx \frac{v*}{\kappa} \ln\left(\frac{h}{z_0}\right) \quad (6-34)$$

In this way it is possible to relate the Chezy coefficient, water depth $h$ and roughness height (bed roughness) $k_s$ and critical velocity $v*$ as follows (Liu 2001):

$$C = \frac{\sqrt{g}}{\kappa} \ln\left(\frac{h}{z_0}\right) \approx \begin{cases} 18 \log\left(\frac{12hv*}{3.3v}\right) & \text{Hydraulically smooth flow; } \frac{v* k_s}{v} \leq 5 \\ 18 \log\left(\frac{12h}{k_s}\right) & \text{Hydraulically rough flow; } \frac{v* k_s}{v} \geq 70 \end{cases} \quad (6-35)$$
where the equations for $z_0$, (6-23) have been utilized. The log used in the equations refers to Briggs logarithm.

**Bottom friction coefficient $f$ versus roughness $k_s$**

Liu (2001) shows derivation of a dimensionless friction coefficient $f$ which corresponds to the Darcy-Weissbach friction coefficient $f_{DW}$ originally derived for pipe flow. Here a different symbol $f$ is used since the derivations do not originate from theory for pipe flows. The derivation is based on examining the forces acting on a grain resting on the bed. The drag force is slightly modified from (6-28) by multiplying the depth averaged velocity $v$ by an empirical coefficient $\alpha$ to take into account the fact that the true velocity near the grain on the bottom is somehow related to $v$. Then it is possible to examine the shear stress $\tau_b$ acting on the grain by saying that the horizontal force is the drag force acting on $A'$ which is the projected area of the grain to the horizontal plane:

$$F_D = \frac{1}{2}\rho C_D A (\alpha v)^2$$  \hspace{1cm} (6-36)

$$\tau_b = \frac{F_D}{A'} = \frac{1}{2}\rho (C_D \alpha^2 \frac{A}{A'}) v^2 = \frac{1}{2}\rho f v^2$$  \hspace{1cm} (6-37)

where $f$ is empirical friction coefficient corresponding the Darcy-Weissbach coefficient in pipe flow.

$$f = (C_D \alpha^2 \frac{A}{A'})$$  \hspace{1cm} (6-38)

Eq. (6-38) is not useful and therefore the following derivation is needed by utilising Eqs. (6-37), (6-15) and (6-31):

$$f = \frac{2\tau_b}{\rho v^2} = \frac{2\rho g R S_o}{\rho C^2 R S_o} = \frac{2g}{C^2}$$  \hspace{1cm} (6-39)

where it has been assumed that flow is uniform. Eq. (6-39) can finally be converted to hydraulically smooth and rough flow conditions by utilising (6-35).


\[
f = \frac{2g}{C^2} \approx \begin{cases} 
0.0555 & \text{Hydraulically smooth flow; } \frac{v \cdot k_s}{v} \leq 5 \\
\frac{\log \left( \frac{12h v_*}{3.3v} \right)}{2} & \text{Hydraulically rough flow; } \frac{v \cdot k_s}{v} \geq 70 
\end{cases}
\]

(6-40)

Friction coefficient equation (6-40) together with Eq. (6-37) provide in some cases a useful way to calculate the bottom shear stress \( \tau_b \).

Colebrook-White equation relating \( f_{WD} \) and \( k_s \) was originally developed for pipe flows and it relates the friction coefficient \( f_{WD} \) with \( k_s \), Reynolds number \( Re \) and \( D \) is hydraulic diameter of the pipe.

\[
\frac{1}{\sqrt{f_{WD}}} = -2 \log \left( \frac{c_1}{Re \sqrt{f_{WD}}} + \frac{k_s}{D} \right) 
\]

(6-41)

where the coefficient \( c_1=2.51 \) and \( c_2=3.71 \) are based on experiments in pipe flows. The same values are used for open channel flow. Moreover, an effective hydraulic diameter \( D_{\text{eff}} \) is used instead real pipe diameter by introducing an empirical correction factor \( f_{PL} \), i.e. \( D_{\text{eff}}=f_{PL}D \) (i.e. \( D_{\text{eff}} \) is used in Eq. (6-41) but in the final equation (6-42) the pipe diameter \( D \) is used):

\[
\frac{1}{\sqrt{f_{WD}}} = -2 \log \left( \frac{c_1}{Re f_{PL} \sqrt{f_{WD}}} + \frac{k_s}{D} \right) 
\]

(6-42)

The first term on the right hand side of (6-42) refers to hydraulically smooth flow and it can be neglected in hydraulically rough turbulent flow. Moreover, in open channel flow the \( D \) is replaced by \( D=4R \) in open channel flow leading to relationship between \( f_{WD} \) and bed roughness \( k_s \) and hydraulic radius \( R \).
\[
\frac{1}{\sqrt{f_{DW}}} = -2 \log \left( \frac{k_s / D}{f_{PL}c_2} \right) = 2 \log \left( 14.84 f_{PL} \frac{R}{k_s} \right) \tag{6-43}
\]

where \( f_{PL} \) is usually 0.827. Eq. (6-43) resembles the equation derived by Liu in (6-40) but with the comment that \( f_{DW} = 4f \) (the factor 2 before log-term raised to exponent 2 causes the conversion factor 4). For wide channels \( R \) is almost the same than \( h \).

Meyer-Peter empirical relationship between \( k_{St} \) and \( d_{90} \)

Assuming that \( k_s = d_{90} \), it is possible to use the Meyer-Peter empirical relationship between \( k_{St} (=1/n) \) and \( k_s \):

\[
k_{St} = \frac{1}{n} = \frac{26}{k_s^{1/6}} \tag{6-44}
\]

6.3.9 Velocity distribution in unvegetated channels

Example 6-1. Consider an unvegetated wide channel with the following input data (see Table 6-1):

**Table 6-1.** Input data for calculation of velocity distribution in unvegetated channel in the case of rippled and flat bed.

<table>
<thead>
<tr>
<th>Input data</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{visc}</td>
<td>\text{1.00E-06}</td>
</tr>
<tr>
<td>\text{d50 g}</td>
<td>\text{3.00E-03}</td>
</tr>
<tr>
<td>\text{h}</td>
<td>1</td>
</tr>
<tr>
<td>\text{SlopeE}</td>
<td>0.0001</td>
</tr>
<tr>
<td>\text{Karman}</td>
<td>4.00E-01</td>
</tr>
<tr>
<td>\text{rho}</td>
<td>1000.0</td>
</tr>
<tr>
<td>\text{Hr}</td>
<td>0.0500</td>
</tr>
<tr>
<td>\text{ks}</td>
<td>0.0600</td>
</tr>
<tr>
<td>\text{ks flat}</td>
<td>0.0075</td>
</tr>
</tbody>
</table>

\( H_r \) = Ripple height, give as input data

\( \text{Bed roughness } = k_s = H_r \) (here)

\( \text{Flat bed } k_s = 2.5 \times d_{50} \)
The example is calculated for two different cases: rippled bed with ripple height $H_r=0.05$ m and it is assumed that $k_s=H_r$ for rippled bed. For flat bed $k_s$ is assumed to be $2.5d_{50}$.

In the calculation of the logarithmic velocity profile it is necessary to know the friction velocity $v_*$ and $z_0$ in equation (6-22a) or (6-22b)

$$v(z) = \frac{v_*}{\kappa} \ln \left( \frac{z}{z_0} \right) \quad \text{(6-22a)}$$

$$\frac{v(z)}{v_*} = \frac{1}{\kappa} \ln \left( \frac{z}{k_s} \right) + 8.5 \quad \text{(6-22b)}$$

Note that (6-22a) and (6-22b) give identical result since there is a relationship between $k_s$ and $z_0$, i.e. (6-22b) gives zero velocity at location

$$z(v=0) = k_s e^{-8.5\kappa} = 0.033k_s = z_0 \quad \text{(for rough flow)} \quad \text{(6-22c)}$$

The calculation of friction velocity is carried out using Eq. (6-15) taking into account that for a wide river or floodplain hydraulic radius $R$ can be replaced by water depth $h$. Bottom slope $S_o$ is replaced by friction slope $S_f$. It is also useful to calculate bottom shear stress $\tau_b$ from Eq. (6-14). Calculation of $z_0$ is carried out using (6-23).
Table 6-2. Calculation of additional variables needed in computation of velocity distribution in unvegetated channel in the case of rippled and flat bed.

<table>
<thead>
<tr>
<th>Calculation of additional variables needed</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>fff</td>
<td>0.039220493 Friction coefficient f from Liu/Eq. (6-40) multiplied by 4</td>
</tr>
<tr>
<td>fff_flatt</td>
<td>0.023007393 Friction coefficient assuming flat bed/D-W eq.</td>
</tr>
<tr>
<td>fDW</td>
<td>0.04089693 Darcy-Weissbach friction coefficient from Eq. (6-43)</td>
</tr>
<tr>
<td>Tau</td>
<td>0.981 Bottom shear stress (\tau_b (N/m^2))</td>
</tr>
<tr>
<td>uss</td>
<td>0.03132092 Friction velocity (u_x (m/s))</td>
</tr>
<tr>
<td>Thick_v</td>
<td>0.000370359 Thickness of the viscous sublayer (m)</td>
</tr>
<tr>
<td>u'ks/Visc</td>
<td>1666.045976 Test if flow hydraulically rough&gt;70</td>
</tr>
<tr>
<td>u'ks_flat/</td>
<td>234.9069965 Transition : 5 &lt; u'ks_flatt/Visc &lt; 70</td>
</tr>
<tr>
<td>zo</td>
<td>0.001668663 Elevation corresponding to zero velocity</td>
</tr>
<tr>
<td>zo_flatt</td>
<td>0.000254003 Elevation corresponding to zero velocity/flat bed</td>
</tr>
<tr>
<td>delz</td>
<td>0.002495828 Interval in numerical integration (200 points)</td>
</tr>
<tr>
<td>delz_flatt</td>
<td>0.002495365 Interval in numerical integration (200 points)</td>
</tr>
</tbody>
</table>

The additional variable shown in Table 6-2 are the thickness of the viscous sublayer for smooth flow (Eq. (6-27)), test criteria \(v_k/s\) for Eq. (6-18) to check the type of flow.

The velocity distribution is shown as numbers in Table 6-3 and in Fig. 6-8a for the whole depth range (0...1 m) and in Fig. 6-8b for the narrow range close to the bottom.

![Velocity distribution in unvegetated channel](image)

Fig. 6-8a. Velocity distribution in rippled and flat bed: water depth \(h=1.0 \text{ m}\), \(S_r=0.0001\).
Fig. 6-8b. Velocity distribution in rippled and flat bed near the bottom: water depth $h=1.0$ m, $S_f=0.0001$.

### Table 6-3.
Velocity distribution in unvegetated channel; water depth $h=1.0$ m, $S_f=0.0001$, ripple height $H_r=k_s=0.05$ m and $k_s=2.5d_{50} = 0.0075$ m for flat bed ($d_{50} =0.3$ mm).

<table>
<thead>
<tr>
<th>Rippled bed/rough</th>
<th>Flat bed/transitional</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>$v(z)/ripp$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.001669</td>
<td>0.000264</td>
</tr>
<tr>
<td>0.004164</td>
<td>0.071613</td>
</tr>
<tr>
<td>0.00666</td>
<td>0.108392</td>
</tr>
<tr>
<td>0.009166</td>
<td>0.133302</td>
</tr>
<tr>
<td>0.011662</td>
<td>0.162177</td>
</tr>
<tr>
<td>0.014148</td>
<td>0.197374</td>
</tr>
<tr>
<td>0.016644</td>
<td>0.180986</td>
</tr>
<tr>
<td>0.019139</td>
<td>0.191038</td>
</tr>
<tr>
<td>0.985259</td>
<td>0.49942</td>
</tr>
<tr>
<td>0.986025</td>
<td>0.499619</td>
</tr>
<tr>
<td>0.997521</td>
<td>0.499917</td>
</tr>
<tr>
<td>0.998017</td>
<td>0.600016</td>
</tr>
<tr>
<td>0.992513</td>
<td>0.500212</td>
</tr>
<tr>
<td>0.996008</td>
<td>0.600409</td>
</tr>
<tr>
<td>0.997504</td>
<td>0.500605</td>
</tr>
<tr>
<td>1</td>
<td>0.500801</td>
</tr>
</tbody>
</table>

As shown in Figs. 6-8a and b and in Table 6-3, the difference between the velocity
distribution in rippled and flat bed is quite large and the difference stems from the very narrow range near the bottom (Fig. 6-8b) where \( v(z) \) increases fast.

Average velocity \( v_A \) can be calculated by numerical integration of the velocity distribution but very accurate results can be obtained using Eq. (6-34).

\[
v_A \approx \frac{v_*}{\kappa} \ln\left( \frac{h}{z_0e} \right)
\]  

(6-34)

In the case of rippled bed the average velocity is \( v_A = 0.42 \) m s\(^{-1}\) and for flat bed \( v_A = 0.57 \) m s\(^{-1}\).

Influence of bottom roughness \( k_s \) on average velocity calculated using Eq. (6-34) is shown in Table 6-4.

<table>
<thead>
<tr>
<th>( h=1 ) m, ( \nu^+ = 0.031321 ) m/s; ( S_f=0.0001 )</th>
<th>( k_s ) (m)</th>
<th>Type of flow</th>
<th>( z_0 ) (m)</th>
<th>( V )-average</th>
<th>( f/Darcy\text{-}W )</th>
<th>VI( \nu ) Eq. (6-32)</th>
<th>( v_A/v_D\nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>1.6</td>
<td>Smooth</td>
<td>3.51203E-06</td>
<td>0.905</td>
<td>0.008605</td>
<td>0.955</td>
<td>0.943</td>
</tr>
<tr>
<td>0.0025</td>
<td>7.8</td>
<td>Transitional</td>
<td>1.18545E-05</td>
<td>0.810</td>
<td>0.011361</td>
<td>0.831</td>
<td>0.974</td>
</tr>
<tr>
<td>0.0005</td>
<td>15.7</td>
<td>Transitional</td>
<td>2.0197E-06</td>
<td>0.769</td>
<td>0.012972</td>
<td>0.778</td>
<td>0.998</td>
</tr>
<tr>
<td>0.01</td>
<td>31.3</td>
<td>Transitional</td>
<td>3.6882E-06</td>
<td>0.721</td>
<td>0.014953</td>
<td>0.724</td>
<td>0.995</td>
</tr>
<tr>
<td>0.002</td>
<td>62.3</td>
<td>Transitional</td>
<td>7.0252E-06</td>
<td>0.671</td>
<td>0.017424</td>
<td>0.671</td>
<td>0.999</td>
</tr>
<tr>
<td>0.004</td>
<td>125.3</td>
<td>Rough</td>
<td>0.00013348</td>
<td>0.620</td>
<td>0.020562</td>
<td>0.618</td>
<td>1.004</td>
</tr>
<tr>
<td>0.008</td>
<td>250.6</td>
<td>Rough</td>
<td>0.00026696</td>
<td>0.566</td>
<td>0.024631</td>
<td>0.564</td>
<td>1.003</td>
</tr>
<tr>
<td>0.016</td>
<td>501.1</td>
<td>Rough</td>
<td>0.00065392</td>
<td>0.512</td>
<td>0.03004</td>
<td>0.611</td>
<td>1.001</td>
</tr>
<tr>
<td>0.032</td>
<td>1002.3</td>
<td>Rough</td>
<td>0.00106784</td>
<td>0.459</td>
<td>0.037448</td>
<td>0.458</td>
<td>0.999</td>
</tr>
<tr>
<td>0.05</td>
<td>1666.0</td>
<td>Rough</td>
<td>0.00166686</td>
<td>0.423</td>
<td>0.043768</td>
<td>0.423</td>
<td>0.998</td>
</tr>
<tr>
<td>0.1</td>
<td>3132.1</td>
<td>Rough</td>
<td>0.003337</td>
<td>0.369</td>
<td>0.057291</td>
<td>0.370</td>
<td>0.996</td>
</tr>
<tr>
<td>0.2</td>
<td>6264.2</td>
<td>Rough</td>
<td>0.006674</td>
<td>0.314</td>
<td>0.078208</td>
<td>0.317</td>
<td>0.993</td>
</tr>
<tr>
<td>0.4</td>
<td>12528.4</td>
<td>Rough</td>
<td>0.013348</td>
<td>0.261</td>
<td>0.113081</td>
<td>0.263</td>
<td>0.990</td>
</tr>
<tr>
<td>0.8</td>
<td>25056.7</td>
<td>Rough</td>
<td>0.026696</td>
<td>0.208</td>
<td>0.177779</td>
<td>0.210</td>
<td>0.988</td>
</tr>
</tbody>
</table>

The results of Table 6-4 show that when flow is smooth the depth-averaged velocity is \( v_A = 0.905 \) m s\(^{-1}\) and for first rough flow case (\( k_s = 0.004 \) m), \( v_A = 0.62 \) m s\(^{-1}\), i.e. about 31 % smaller. The results of Table 6-4 show how crucial it is to be able to evaluate the bed roughness correctly. Comparison of the depth-averaged velocity calculated from velocity distribution is also compared to velocity given by the Darcy-Weissbach equation (6-32).
The results given in the last three columns in Table 6-4 show that the difference between the values is quite small. Calculation of the Darcy-Weissbach friction factor $f_{\text{DW}}$ is carried out using Eq. (6-43) and assuming that $f_{\text{PL}}=0.827$ and $R$ is replaced by $h$. The results are very close to each other for rough flow conditions, i.e. conditions prevailing in natural channels.

### 6.3.10 Exercise

Calculate velocity distribution $v(z)$ (show also as a graph) in unvegetated channel using the following input data. Assume that the bottom is rippled and $k_s = H_r$.

<table>
<thead>
<tr>
<th>Input data</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>viscosity</td>
<td>$1.00 \times 10^{-6}$</td>
</tr>
<tr>
<td>$d_{50, g}$</td>
<td>$4.00 \times 10^{-3}$</td>
</tr>
<tr>
<td>$h$</td>
<td>1.2</td>
</tr>
<tr>
<td>SlopeE</td>
<td>0.0008</td>
</tr>
<tr>
<td>Karman</td>
<td>$4.00 \times 10^{-1}$</td>
</tr>
<tr>
<td>$H_r$</td>
<td>0.0500</td>
</tr>
<tr>
<td>$k_s$</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

Calculate average velocity by integrating numerically the velocity distribution over the depth $h$. Calculate also average velocity using Eq. (6-34) and compare the result with average velocity calculated using the Darcy-Weissbach equation.