Biosensors and Instrumentation: Tutorial 2 Solutions

1. One of the most straightforward methods of monitoring temperature is to use the thermal variation of a resistor.

1.1. Suggest a possible problem with the use of a standard Wheatstone bridge setup in a remote application like an implanted sensor?

*The other resistors used in the bridge will also have some temperature dependence which will be confounded with that of the RTD if care is not taken. It would be very hard to control temperatures in an implanted device like that.*

1.2. Can you suggest a possible solution?

*It is unlikely but if the temperature dependence of the other resistors can be made to be much much smaller than the measured device then this could almost be ignored.*

1.3. The alternative might be to locate the resistor at the remote sensing position while the instrumentation is in a stable environment. Why does this lead to a problem?

*The resistance of the electrical connections to the RTD become a problem. These could be balanced at one particular temperature but they may also have some temperature dependence that will cause an error in the measurement.*

1.4. Why does a Kelvin measurement improve measurements of small resistances?

*Current and voltage are separated in a four-terminal Kelvin measurement meaning that the resistance of the electrical connections should not impact on the measured resistance.*

2. The circuit shown in figure 1 can be used to compensate for the error in the output voltage which results from the fact that the “cold junction” is not at 0 °C. There are two parameters which can control this compensation (assuming all of the resistors in the Wheatstone bridge are equal when the cold bridge temperature is 0 °C), which are the voltage used to power the bridge and the position of the potentiometer. If we have the following parameters suggest values for \( V_{in} \), and the ratio of \( R_{P1} \) to \( R_{P2} \) to correct for an observed error in the thermocouple output EMF of –10mV when the cold junction is at room temperature (20 °C):

- \( R_{RTD} = 100 \, \Omega \) @ \( T_{junction} = 0 \, °C \)
- \( \alpha_{RTD} = 0.00385 \, \Omega \, \Omega^{-1} \, K^{-1} \)
- \( R = 100 \, \Omega \) (Wheatstone bridge resistor)
- \( R_{PA} = R_{P1} + R_{P2} = 100 \, k\Omega \)

*The first step in solving this is to determine the output of the Wheatstone bridge, which gives us the temperature dependent voltage which can then be used to correct for the error due to the cold junction temperature. The resistance \( R_{RTD} \) at 20 °C is:*

\[
R_{RTD} = R_0[1 + \alpha(T - T_0)] = 107.7 \, \Omega
\]

1 Hint for Q2, if \( R_{RTD} \) is greater than \( R \) then think about the size and direction of the voltage output from the Wheatstone bridge then about how that adds/subtracts from the EMF from the thermocouple and the cold junctions in order to correct for any error. It might be useful to redraw the circuit with voltage sources replacing these components.
Figure 1. Thermocouple compensation circuit

Then, the output of the Wheatstone bridge will be:

\[ V_{bridge} = V_{in} \left( \frac{R}{R + R_{RTD}} - \frac{R}{R + R} \right) \]

Which reduces to:

\[ V_{bridge} = V_{in} \left( \frac{R - R_{RTD}}{2(R + R_{RTD})} \right) \]

If we choose \( V_{in} \) to be a sensible value like 5 V, then this will be (at 20 °C):

\[ V_{bridge} = -92.7 \text{ mV} \]

Which is helpfully larger in magnitude than the thermocouple voltage error of -10 mV stated in the question.

The next step is to determine the ratio of \( R_{P1} \) and \( R_{P2} \) required to correct for the error. The corrected output EMF will be:

\[ EMF = V_{thermocouple} - V_{RP1} \]

While the thermocouple output will be:

\[ V_{thermocouple} = EMF + V_{error} \]

So all we need to do is arrange for \( V_{RP1} = V_{error} = -10 \text{ mV} \)

Sorry for labouring this point somewhat I just want to try to make it clearer what’s going on. Potentially a similar setup could be used to correct for other temperature dependent errors in completely different types of sensor. Anyway, now we can state that the voltage on the upper portion of the potentiometer is:

\[ V_{RP1} = V_{bridge} \left( \frac{R_{P1}}{R_{P1} + R_{P2}} \right) \]

We can then rearrange this in terms of \( R_{P1} \) and all the other terms are known so:

\[ R_{P1} = \frac{V_{RP1}}{V_{bridge}} (R_{P1} + R_{P2}) \approx 10.8 \text{ kΩ} \]

and therefore \( R_{P2} = 89.2 \text{ kΩ} \).
3. The basic formula for resistance is $R = \rho \frac{L}{A}$. From this we can work out the fractional change in resistivity for a strain gauge:

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} - \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

The first term there deals with the change in the material resistivity due to applied strain. The piezoresistive coefficient $\pi$ is defined as:

$$\pi = \frac{\Delta \rho}{\rho E \varepsilon}$$

Where $E$ is the Young’s modulus of the material and $\varepsilon$ is the strain ($\varepsilon = \Delta L/L$).

Poisson’s ratio $\nu$ is another material parameter that defines the change in the transverse strain due to an applied axial strain, for our resistor we can say that:

$$\frac{\Delta A/A}{\Delta L/L} = -2\nu$$

3.1. Combine all of this information with the equation for the gauge factor of a piezoresistor (given below) to produce an expression for $GF$ in terms of $\nu$, $\pi$ and $E$.

$$GF = \frac{\Delta R}{\varepsilon R_G}$$

*Going back to the piezo resistive coefficient:*

$$\frac{\Delta \rho}{\rho} = \pi E \varepsilon$$

*Then using the formula for gauge factor:*

$$GF = \frac{1}{\varepsilon} \left( \frac{\Delta \rho}{\rho} - \frac{\Delta A}{A} + \frac{\Delta L}{L} \right)$$

*Finally we remember that strain is $\varepsilon = \Delta L/L$ and the formula for Poisson’s ratio given above and substitute where appropriate:*

$$GF = 1 + 2\nu + \pi E$$

3.2. In most metals the poisson ratio is between 0 and 0.5, while the effect of $\pi$ is negligible, what does that mean for the possible range of values for $GF$?

*With no significant piezoresistive effect the gauge factor will vary between 1 (for $\nu = 0$) and 2 (for $\nu = 1$).*

3.3. Polycrystalline silicon can have a gauge factor as high as 40. What is the source of this?

*Semiconductors all have significant piezoresistive effects meaning that $\pi$ dominates in the GF equation. The source of this is the effect of strain on the mobility of charge carriers in the semiconductor.*

4. A platinum based strain gauge with an unstrained resistance ($R_G$) of 1k$\Omega$ has a gauge factor ($GF$) of 2. The formula for gauge factor is covered in the previous question.
4.1. If the gauge is securely fixed to a measured structure undergoing a strain ($\varepsilon$) of 10%, what will be the maximum change in resistance ($\Delta R$)?

$$\Delta R = GF\varepsilon R_G = 2 \times 0.1 \times 1000 = 200 \ \Omega$$

This is obviously quite large and reflects the rather unrealistically high strain value.

4.2. You have 4 gauges to measure a structure with a maximum strain of 1%. How would you arrange the gauges in a Wheatstone bridge in order to maximise the output voltage and what will be the output voltage range?

The gauges will be applied to the structure so that two are in compression ($R_{C1}$, $R_{C2}$) and two are in tension ($R_{T1}$, $R_{T2}$) these will be arranged so that they are opposite each other as in figure T1. Assuming the same gauge factor as before $\Delta R$ will be 20Ω. Rather stupidly I wrote this without stating a bridge input voltage but if this is 5V then on one side of the bridge the divider voltage will be:

$$V_1 = V_B \frac{R_{C1}}{R_{C1} + R_{T1}} = 5 \times \frac{980}{980 + 1020} = 2.45 \ \text{V}$$

While on the other side it will be:

$$V_2 = V_B \frac{R_{T2}}{R_{C2} + R_{T2}} = 5 \times \frac{1020}{980 + 1020} = 2.55 \ \text{V}$$

So $V_{\text{out}}$ will be 100mV.

![Wheatstone bridge arrangement using 4 strain gauges](image)

4.3. Assuming it’s only possible to attach two of the 4 gauges to the structure under test, how would you do this and what effect would it have on the sensitivity?

Attach one in compression ($R_{C1}$) and one in tension ($R_{T1}$) while the other two resistances will be standard resistors with values chosen to balance the unstrained gauges. The sensitivity will be halved so that the output for a strain of 1% will be 50mV.

4.4. How could you add to the Wheatstone bridge circuit in order to allow small mismatches in the gauge resistors to be corrected?

A variable resistor could be added to one arm of the circuit. This could be adjusted to give a zero output when the gauges are unstrained.
5. Piezoelectric acoustic wave devices are now widely used in RF filters in mobile communications and in passive RF-ID tags. Surface acoustic wave filters can also be adapted into sensitive detectors. The frequency of the acoustic wave is proportional to the wave velocity in the material.

5.1. Suggest how this could be used in a biosensor.

*Biosensing molecules can be attached to the surface of the SAW sensor, changing the mass that is being moved by the wave, effectively changing the transmission velocity. Subsequent attachment of the chemical/marker being sensed will further change this and the change can be detected.*

5.2. What factors can also affect the frequency?

*Temperature and applied strains can also affect the SAW frequency, by altering the wave velocity or by slightly changing the distance between the interdigitated electrodes.*

5.3. How can these be corrected for? Sketch a SAW sensor setup that could avoid these problems which only uses one driven IDE.

*The RF pulse is fed to the central IDE which sends out SAW pulses in opposite directions. These are reflected back and the difference in timing between the pulse from the sensing and reference sides contains information about the status of the sensing area. Changes due to temperature or applied strain should be common for both sides of the sensor and can be cancelled out.*

5.4. Why are “Rayleigh wave” type SAW devices unsuitable for use in liquid environments?

*Rayleigh waves have an amplitude that is normal to the surface of the SAW device and are heavily damped by contact with dense fluids. This could prevent the propagation of the SAW waves from the input transducer to the input.*

6. The capacitive bridge circuit shown in figure 2 is used to convert changes in the capacitance of a MEMS accelerometer with a differential sensing setup. It has the following output voltage amplitude, when driven with an ac voltage with amplitude $V$:

$$V_{out} = V_2 - V_1 = V \frac{x}{2d}$$

When $C_2 = C_4 = C_0$, $C_1 = C_{w1}$ and $C_3 = C_{w2}$ where

$$C_{w1} = \frac{C_0}{1 - \frac{x}{d}} = C_0 + \Delta C$$

and

$$C_{w2} = \frac{C_0}{1 + \frac{x}{d}} = C_0 - \Delta C$$

(1)

(2)
6. Using equations (1) and (2), define the change in capacitance $\Delta C$. Is this the same for $C_{w1}$ and $C_{w2}$?

Rearrange equation (1):

$$C_0 = (C_0 + \Delta C)(1 - \frac{x}{d})$$

$$C_0 = C_0 + \Delta C - C_0 \frac{x}{d} - \Delta C \frac{x}{d}$$

$$C_0 \frac{x}{d} = \Delta C \left(1 - \frac{x}{d}\right)$$

$$\Delta C = \frac{C_0 \frac{x}{d}}{1 - \frac{x}{d}}$$

Then if we assume that $x \ll d$ then the difference will become:

$$\Delta C = C_0 \frac{x}{d}$$

Similarly we can rearrange equation (2):

$$\Delta C = \frac{C_0 \frac{x}{d}}{1 + \frac{x}{d}} \approx C_0 \frac{x}{d}$$

So, there’s a small difference between the $\Delta C$ values for the two differential capacitances if the displacement is significant.

6.2. If we can arrange for this to be a full bridge where $C_1 = C_4 = C_{w1}$ and $C_2 = C_3 = C_{w2}$ what will the equation for the output voltage be in terms of the displacement $x$?

The equations for the output voltages are as follows:

$$V_1 = V \frac{C_2}{C_2 + C_4} = V - \frac{C_0}{1 + \frac{x}{d}} + \frac{C_0}{1 - \frac{x}{d}} = V \left(2 - \frac{x}{2d}\right)$$

$$V_2 = V \frac{C_1}{C_1 + C_3} = V - \frac{C_0}{1 + \frac{x}{d}} + \frac{C_0}{1 - \frac{x}{d}} = V \left(2 + \frac{x}{2d}\right)$$
and we know that \( V_{out} = V_2 - V_1 \) which becomes:

\[
V_1 - V_2 = V \left( 2 + \frac{x}{2d} \right) - V \left( 2 - \frac{x}{2d} \right) = V \frac{x}{d}
\]

6.3. If the area of one of the differential capacitors is 400×400\( \mu \)m and the separation \( d = 1 \mu \)m calculate the output voltage from the bridge when the deflection \( x = 10 \text{nm} \) and \( V = 5 \text{V} \).

As the output is independent of \( C_0 \) the output will simply be \( 5 \times 10 / 1000 = 50 \text{ mV} \)

6.4. If there are parasitic capacitances \( C_p \) on the two output terminals what is the equation for the output voltage? (hint, use the \( \Delta C \) version of the working capacitor equations)

The output voltages now become:

\[
V_1 = V \frac{C_2}{C_2 + C_4 + C_p} = V \frac{C_0 - \Delta C}{(C_0 - \Delta C) + (C_0 + \Delta C) + C_p} = V \frac{C_0 - \Delta C}{2C_0 + C_p}
\]

and:

\[
V_2 = V \frac{C_1}{C_1 + C_3 + C_p} = V \frac{C_0 + \Delta C}{(C_0 + \Delta C) + (C_0 - \Delta C) + C_p} = V \frac{C_0 + \Delta C}{2C_0 + C_p}
\]

then the output of the bridge will be:

\[
V_{out} = V_2 - V_1 = V \left( \frac{C_0 + \Delta C}{2C_0 + C_p} - \frac{C_0 - \Delta C}{2C_0 + C_p} \right) = V \frac{2\Delta C}{2C_0 + C_p}
\]

6.5. What are the possible sources of parasitic capacitance in MEMS sensors?

These could include the capacitance between a suspended MEMS structure and a conducting substrate, parasitics between electrical interconnect and the substrate and contributions from active devices in the instrumentation, such as input capacitance of FET based op-amps.

6.6. The main contribution to the parasitics is from the bond pads used to connect to the bridge. The pads are 80×80\( \mu \)m and are separated from the grounded silicon substrate by a 2\( \mu \)m thick dielectric (\( \varepsilon_R = 7 \)). Calculate the value of \( C_p \).

\[
C_p = \frac{\varepsilon_0 \varepsilon_R A}{d}
\]

which comes out to be around 0.198pF

6.7. Recalculate the output voltage for the same deflection as question 1.2 including the effect of the parasitics.

Using the information above we can estimate \( C_0 = 1.42 \text{pF} \). We will assume that \( x \) is small enough compared to \( d \) to approximate \( \Delta C \) as:

\[
\Delta C = C_0 \frac{x}{d}
\]
So $\Delta C = 14.2 \text{ fF}$ and $V_{out} = 0.04673V$. It’s not a huge effect (<10%) but if $C_p$ was ten times larger (~2pF), which isn’t unheard of then the output would only be about 0.03V, a change of 40%.

7. The alternative to the capacitive bridge for displacement sensing is current measurement using the circuit in figure 3.

\[ i = j\omega V C_0 \left( \frac{1}{1 - \frac{x}{d}} - \frac{1}{1 + \frac{x}{d}} \right) \]

\[ i = j\omega V C_0 \left( \frac{2 \frac{x}{d}}{1 - \left( \frac{x}{d} \right)^2} \right) = j2\omega V C_0 \left( \frac{x}{d} \frac{d^2 - x^2}{d^2} \right) \]

\[ i = j2\omega V C_0 d \left( \frac{x}{d^2 - x^2} \right) \]
7.3. If this circuit is used to measure a micromachined accelerometer with nominal gap \( d = 2.5 \, \mu m \) and area \( A = 1.2 \, mm \times 1.2 \, mm \) calculate the output current when the displacement is 0.46 \( \mu m \) and the device is driven with an ac voltage \( V = 0.1 \, V, f = 100 \, kHz \).

*First calculate \( C_0 = 5.1pF \) then slot all the numbers into the equation to get \( i = 0.12\mu A \).*

7.4. If a current-follower op-amp circuit is used to amplify and convert this current, what value of resistor is required to give a 1V p-p voltage output?

*For a current follower \( V_{out} = -iR \), so to get 1V from 0.12\( \mu A \), \( R \approx 8.2M\Omega \).*

8. Switched capacitor circuits may be more appropriate for use in IC based sensor interfaces.

8.1. Give as many reasons as you can come up with why resistors are a problem in integrated electronics, particularly if an amplifier circuit such as that in question 3.4 is required. What are the advantages of capacitors?

**Disadvantages of resistors for biasing integrated amplifiers:**
- Large value resistors are physically large, sheet resistance usually defined in process.
- Resistor matching can be poor
- Resistors are thermal noise sources, especially large values
- Resistor values have significant temperature dependence

**Advantages of Capacitors:**
- Capacitors can be made smaller than resistors
- Capacitor ratios can be easily set
- Capacitors are not thermal noise sources
- Capacitors temperature dependence is low

8.2. The circuit shown in figure 4 is controlled by two non-overlapping clock signals such that when \( s1 \) and \( s3 \) are closed, \( s2 \) is open and vice-versa. With the help of sketches of the circuit during the two different phases, describe the operation of the amplifier and state the transfer function.

![Switched Capacitor Amplifier](image-url)
Figures T3 and T4 show the two different switch positions. In phase 1 (T1), with S1 and S3 closed C₁ charges to Vᵢₜ so \( Q_{C₁} = V_{in} C₁ \). Meanwhile C₂ is discharged. There is a possible problem with this that one input of the op-amp is floating not controlled so it’s perhaps not practical. It may be that \( V_{out} \) will saturate in this case or will it go to 0V? Anyway, when the switches change to phase two (T2) C₁ will discharge and the current flow will charge up C₂ but the direction of this will mean that:

\[
V_{out} = \frac{Q_{C₂}}{C₂} = -\frac{Q_{C₁}}{C₂} = -V_{in} \frac{C₁}{C₂}
\]

So the transfer function is:

\[
\frac{V_{out}}{V_{in}} = -\frac{C₁}{C₂}
\]

and this is an **Inverting** switched capacitor amplifier.

![Figure T3: SC circuit in phase 1, S1 & S3 closed, S2 open.](image)

![Figure T4: SC circuit in phase 2, S2 closed, S1 & S3 open.](image)

8.3. Figures 5 and 6 show two more switched capacitor circuits driven by non-overlapping clock signals \( \Phi₁ \) and \( \Phi₂ \). What is the difference between the transfer functions of each?

**SC amplifier number 1** will have the following transfer function:

\[
\frac{V_{out}}{V_{in}} = \frac{C₁}{C₂}
\]

So it is a non-inverting amplifier with a gain set by the capacitor ratio.

**In SC circuit number 2** we need to begin by assuming that at the start of cycle \( \Phi₁ \) both capacitors are discharged. Then, during \( \Phi₁ \), \( C₁ \) will be charged to \( V_{in} \) as the other end is at a virtual earth. This charge will also accumulate on \( C₂ \) with a result that \( V_{out} = -Q_{C₂}/C₂ \). So the transfer function will be:
\[ \frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{C_1}{C_2} \]

So this second circuit is an inverting amplifier with a gain set by the capacitor ratio. In cycle \( \Phi_2 \) both capacitors are discharged back to zero 0V.

Figure 5. SC amplifier number 1

Figure 6. SC amplifier number 2