

# Which Inequality?

## The Inequality of Endowments Versus the Inequality of Rewards\*

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### Abstract

Society often allocates valuable resources - such as prestigious positions, salaries, or marriage partners - via tournament-like institutions. In such situations, inequality affects incentives to compete and hence has a direct impact on equilibrium choices and hence material outcomes. We introduce a new distinction between inequality in initial endowments (e.g. ability, inherited wealth) and inequality of what one can obtain as final rewards (e.g. prestigious positions, money). We show that these two types of inequality have opposing effects on equilibrium behavior and wellbeing. Greater inequality of rewards tends to harm most people – both the middle class and the poor – who increase their effort. In contrast, greater inequality of endowments can benefit the middle class. Thus, the correctness of our intuition about the implications of inequality is hugely affected by the type of inequality considered.

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# 1 Introduction

Perhaps there is no other economic debate older than that over inequality. While most people agree that some reduction of inequality is desirable, there is no consensus over what is meant by equality, nor over what should be equalized (see Sen (1980), Dworkin (1981a,b), Phelps Brown (1988), Roemer (1996), and many others). For many economists, the second fundamental welfare theorem separates distributional issues from the analysis of efficiency. As a result, the bulk of the current literature on inequality concentrates either on the measurement of inequality, or on the fairness of particular resource distributions.

Here, we address the issue of inequality from a purely economic perspective. We assume a society where individuals differ in terms of initial endowments, whether it is innate ability, education received or inherited wealth, and where these endowments are private information. Further, the rewards that individuals receive as a result of their performance are assigned by a tournament. A fixed set of rewards, that could represent cash prizes, places at a prestigious university, attractive jobs, desirable spouses, social esteem, monopoly rents or any combination of these, vary in terms of their desirability. Individuals make a simultaneous decision about how to divide their endowments between performance in the tournament and private consumption or leisure. Then each individual is given a reward according to his rank in the distribution of performance: first prize is given to first place, second prize to second place, and so on.

Such a tournament creates important positional externalities, as to obtain a top reward one must occupy a top position, and by doing so one excludes others from that position and hence that reward. As observed by Cole, Mailath and Postlewaite (1992) (see also Postlewaite (1998)), this induces competitors to behave as though they had a desire for high relative position, such as in Frank's (1985) classic model of status. In turn, this leads to equilibrium effort being inefficiently high and equilibrium utility being inefficiently low. Crucially, these externalities also imply that the equilibrium choice of effort and equilibrium utility depend on both the initial distribution of endowments and the distribution of rewards. Therefore, there is no need to appeal to any notion of justice for inequality to matter. It matters because what others have affects the job one gets, the wage one is paid and the amount of leisure one takes.

In particular, the shape and the range of the distributions of endowments and rewards themselves determine the marginal return to effort. Thus, changes in the level of inequality of either distribution can affect the equilibrium behavior and utility even of those individuals who see neither a change in their own endowment nor in reward. Further, we find that changes in the inequality of endowments have the opposite effect to changes in the inequality of rewards. A decrease in the inequality of competitors' endowments raises the return to effort as it is easier to overtake one's rivals. This leads to higher effort for low and middle ranking agents. Furthermore, equilibrium utility falls at middle and high ranks and even those with higher endowments can be worse off in the less unequal and hence more competitive distribution. However, a decrease in

the inequality of rewards implies there is less difference between a high prize and a low one. This leads to a reduction in incentives and a decrease in equilibrium effort for low and middle ranking competitors, and an increase in their equilibrium utility. Indeed, under some conditions, even stronger welfare effects are possible - namely that reduced inequality of rewards can make all better off.

Simply put, in the tournament model we consider, a reduction in inequality of rewards can benefit most of society, but lower inequality of endowments can harm the majority. Thus, the inequality of rewards has a much better fit with our intuition about the effects of inequality than the inequality of endowments.

In such a model, even policy interventions such as lump-sum taxes and transfers will have an impact on incentives as they change either the distribution of endowments or of rewards. In fact, there are two distinct effects from any changes in the level of inequality. The first, which we call the direct effect, is simply that under a less unequal distribution of endowments or rewards lower ranked individuals will have greater endowments or rewards respectively. However, in either case, there is also the second effect, which we call the incentive or social competitiveness effect. Crucially, the incentive effect of a decrease in inequality of endowments is positive and opposite of that of a decrease in the inequality of rewards, which decreases incentives. This incentive effect is created by the competitive externalities present in our tournament model. So, in their absence such as in more conventional neoclassical models, there are only the direct effects so that reward and endowment inequality would appear to have similar results. This is possibly why the distinction between rewards and endowments has not been made before.

We further contribute to the modelling of inequality by demonstrating the importance of the method of tracking individuals when endowments change. There are two ways of doing this: compare choices and outcomes at a given level of endowment or at a given rank in the distribution of endowments. As Hopkins and Kornienko (2009) point out and as we show here, the two methods of indexing lead to seemingly contradictory results: lower inequality of endowments leads to higher utility at a given low rank, but lower utility at a given low endowment. However, since in a less unequal distribution low-ranked individuals tend to have higher endowments, these are simply two different ways of looking at the same results.

In summary, our contribution is five-fold. First, we show that, in the tournament model we consider here, inequality can have a direct impact on material outcomes, and thus can be examined using positive methods of economic analysis. Second, we identify two different types of inequality, and examine them within the same model. Third, by employing novel techniques, we show that the two types of inequality often have opposite effects on material outcomes. Fourth, we contrast the results obtained using two different indexing methods. Finally, we argue that tournament models help us to understand different types of social inequality and, thus, help to answer the normative question - which inequality should we care about.

## 1.1 Related Literature

Why should we assume that rewards are determined by tournaments rather than by competitive markets? An important reason is empirical. Tournament-like mechanisms are used in practice to determine university admissions, entry into certain professions and promotions and pay within firms. Second, relative position seems to matter for welfare. There is now a significant body of research that suggests that indicators of wellbeing – such as job satisfaction (Brown et al., (2008)), health (Marmot et al. (1991), Marmot (2004)) and overall happiness (Easterlin (1974)) – are strongly determined by relative position. That is, a highly ranked individual in a poor country can have greater health and happiness than a low ranked individual in a richer country, even though the latter has greater material prosperity. There are two leading hypotheses to explain these empirical findings. The first, pioneered in modern economics by Duesenberry (1949) and Frank (1985), is that people have an intrinsic concern for relative position or status. The second hypothesis is due to the fundamental insight of Cole, Mailath and Postlewaite (1992) that many of life’s crucial rewards are allocated by tournament-like mechanisms, and this induces the appearance of preference for status.

By analyzing a tournament model, clearly we favor the second rationale for why welfare depends on relative position. Yet, equally, our present analysis of the effects of inequality would be also applicable to a model of intrinsic relative concerns. Broadly consistent with our current results, Becker, Murphy and Werning (2005) find that, in a model of status, agents would willingly take lotteries that would increase what we would call here the inequality of endowments. Inequality of endowments in status models is also explored in Hopkins and Kornienko (2004, 2009). The crucial difference is that here we also consider the inequality of rewards, as well as employing a more general specification when considering endowments. Thus, our results concerning the effects of changes in the level of inequality of endowments are generalizations of those in Hopkins and Kornienko (2004, 2009). However, the results on the inequality of rewards, and the idea of contrasting them with the results on inequality of endowments, which we see as the main contribution of the current paper, are entirely novel.

The literature on tournaments and contests is extensive. As Konrad (2009) points out in a survey, increased heterogeneity amongst competitors and decreased spread of prizes are both known to reduce equilibrium effort.<sup>1</sup> The technical contribution here is to consider very general comparative statics for large populations of competitors. The use of rank-order tournament models to study non-market allocation of resources was pioneered by Cole, Mailath and Postlewaite (1992, 1995, 1998), followed by Zenginobuz (1996) and Fernández and Galí (1999). However, their focus of interest is not inequality but a comparison of different institutions for assigning rewards. Two further papers are technically particularly close to our work, yet they also look at different issues. Moldovanu and Sela (2006) consider what would be the optimal contest design from

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<sup>1</sup>Much of this literature concentrates on games in which the mechanism that awards prizes is assumed to be at least partly stochastic. What we model here in a contrast could be called a perfectly discriminating rank order tournament or contest.

the perspective of a contest designer who aimed to maximize either the expected total effort or the expected highest effort from contestants. Hoppe, Moldovanu and Sela (2009) generalize this approach to a two-sided matching tournament problem.

One important assumption of our tournament model is that there is a fixed distribution of indivisible rewards. The justification for this is that in reality there are many desirable things, jobs, places at university, marriage opportunities, that do differ in quality and which are not divisible. A subtle criticism is that even if rewards are indivisible, they might be assigned by prices rather than performance, which might improve efficiency. This possibility is analyzed in a different literature where workers are matched to (indivisible) jobs by an endogenous wage schedule. For example, Costrell and Loury (2004) and Suen (2007) have considered changes in the distribution of ability of workers and in the quality of jobs. There is no incentive effect as there is no choice of effort by workers and all outcomes are Pareto efficient, in distinct contrast to the situation we model. Nonetheless, the shape of the distributions of ability and of jobs affects the distribution of wages. That is, changes in the level of inequality can have a material effect on outcomes even if there is a price mechanism.<sup>2</sup>

We also argue that our distinction between endowments and rewards to be novel in that it differs from the most common existing concepts of equality on three levels. First, we argue that equality of rewards and endowments are logically separate from equality of opportunity. Here, as rewards are determined solely by performance, agents always face equality of opportunity, yet the levels of reward and endowment equality vary.<sup>3</sup> Existing merit-, desert- or effort-based theories of justice assume that those who work more, or have greater merit, should have greater rewards (see Konow (2003) for a survey), however, there seems to be little discussion of the fact that the reward schedule could vary even in the presence of equality of opportunity. Talent could vary widely, but the most talented could receive a monetary reward only slightly greater than the least talented. Alternatively, small differences in talent could lead to big differences in outcomes. Second, in the distributive justice literature (see Rawls (1971), Dworkin (1981b), Roemer (1996, 1998) among others), one often encounters the question of equality of “resources” (wealth, but also possibly education or talent). However, these works make no distinction about timing or causation, in the sense that there is no distinction made between what one has initially (endowment) and what one is able to obtain (reward). Third, equality of rewards should not be confused with equality of welfare or equality of outcomes. In this model at least, the welfare of an individual depends jointly on a set of outcomes that includes her endowment, her choice of effort as well as her reward.

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<sup>2</sup>More technically, inequality of endowments and inequality of rewards will have opposing effects regardless of whether matching between competitors and jobs or rewards is done under transferable utility or non-transferable utility. See Hopkins (2005) for a comparison of the two cases.

<sup>3</sup>The equality of opportunity we consider here is non-discriminatory, or “formal” in the sense of Roemer (1996, p. 163), and “competitive” in the sense of Lloyd Thomas (1977) and Green (1989). We discuss the relation of our work to previous literature on equality in greater detail in the working paper version of this paper.

## 2 The Model

In this section, we develop our model, where a large population competes in a tournament-like market to obtain rewards or prizes. We have in mind three prime examples. The first is students competing for places at college. The second is a market for jobs. For example, students in the final year of graduate school seek faculty positions at universities. The third is a marriage market, where singles attempt to attract desirable potential spouses. These three situations are modelled as tournaments by Fernández and Galí (1999), Hopkins (2005) and Cole, Mailath and Postlewaite (1992) respectively. We will use the terminology of “contestants” competing for rewards. Contestants have to make a decision on how to allocate their initial endowment between private consumption and visible performance that acts as a signal of underlying ability. Each contestant is then awarded a reward or prize. These are awarded assortatively with the best performer being awarded the top prize, the median performer the median prize and so on downward with the worst performer receiving the last prize.

We assume a continuum of contestants. They are differentiated in quality with contestants having differing endowments  $z$  with endowments being allocated according to the publicly known distribution  $G(z)$  on  $[\underline{z}, \bar{z}]$  with  $\underline{z} \geq 0$ . The level of each contestant’s endowment is her private information. The distribution  $G(z)$  is twice differentiable with strictly positive density  $g(z)$ . A contestant’s level of endowment  $z$  has possible interpretations such as her wealth or an ability parameter that determines maximum potential performance.<sup>4</sup> In particular, contestants must divide their endowments between visible performance  $x$  and private consumption or leisure  $y$ .

There is also a continuum of prizes or rewards of value  $s$  whose publicly known distribution has the twice differentiable distribution function  $H(s)$  on  $[\underline{s}, \bar{s}]$  and strictly positive density  $h(s)$ . While the rewards could simply be in cash, this is not necessarily the case. In the context of the academic job market,  $s$  could be interpreted as prestige or reputation of a university, in the marriage market,  $s$  could be a measure of attractiveness to the opposite sex. After the contestants’ choice of performance, rewards will be awarded assortatively, so that the contestant with the highest performance  $x$  will gain the prize with highest value  $\bar{s}$ . More generally, the rank of the prize awarded will be equal to a contestant’s rank in terms of performance.

We have two ideas in mind why rewards might be assigned in such a way. First, such mechanisms are often used in situations such as college admissions to promote a form of equality of opportunity. For example, if  $z$  represents ability and  $x$  represents academic performance, then the highest rewards go to contestants with the highest performance which in the equilibrium we consider will be those of highest ability.<sup>5</sup> Second, the other

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<sup>4</sup>For example, suppose all contestants are endowed with the same amount of time that can be used for production or leisure. Then, let  $z$  be productivity per hour and a contestant devoting a proportion  $x/z$  of time to production will have performance  $x$ .

<sup>5</sup>Fernández and Galí (1999) show that such mechanisms can be more efficient than markets in assigning educational opportunities when capital markets are imperfect.

side of the market could consist of people, potential spouses or employers, rather than inanimate prizes. These potential partners would have to choose between contestants. But it is easy enough to specify suitable preferences for the partners such that the end result in equilibrium would be the same: the best performing contestant obtains the best match.<sup>6</sup> Here, we assume that such partners are interested in a contestant's performance  $x$  mostly in terms of what it signals about his underlying endowment of ability  $z$ .

A contestant's endowment  $z$  can be employed in performance  $x$  or private consumption  $y = z - x$  (that is, the rate of conversion between  $x$  and  $y$  is normalized to one). All contestants have the same utility function

$$U(x, y, s) = U(x, z - x, s). \quad (1)$$

We assume that utility is increasing in all three arguments, performance  $x$ , private consumption  $y$  and the reward  $s$ . That is, there is some private benefit to performance, for example, private satisfaction from studying.<sup>7</sup> While it is possible to divide one's endowment between  $x$  and  $y$ , the only way to obtain a reward  $s$  is to take part in the tournament.

We assume a series of standard conditions on the utility function that will enable us to derive a monotone equilibrium and clear comparative statics results. (i)  $U$  is twice continuously differentiable (smoothness); (ii)  $U_x(x, y, s) > 0$ ,  $U_y(x, y, s) > 0$ ,  $U_s(x, y, s) > 0$  (monotonicity); (iii)  $U_{xy}(x, y, s) > 0$ ,  $U_{ys}(x, y, s) \geq 0$  and  $U_{xs}(x, y, s) \geq 0$  (complementarity); (iv)  $U_{ii}(x, y, s) \leq 0$  for  $i = x, y, s$  (own concavity); (v)  $U_x(x, z - x, s) - U_y(x, z - x, s) = 0$  has a unique solution  $x = \gamma(z, s)$  and whenever  $x \geq \gamma(z, s)$  it holds that  $U_{xs}(x, z - x, s) - U_{ys}(x, z - x, s) \leq 0$ . This last condition seems somewhat complicated but it is automatically satisfied if utility is additively or multiplicatively separable in  $s$ . Note also that it implies a competitor would choose a positive performance  $x$  even when there are no competitive pressures.

It is natural, perhaps, to think of a competitor's type as her level of endowment. However, given an endowment distribution  $G(z)$ , an agent with endowment  $\tilde{z}$  has rank  $\tilde{r} = G(\tilde{z})$  and it is just as valid to think of her type as being  $\tilde{r}$  as much as it is  $\tilde{z}$  (recall that we assumed that  $G(\cdot)$  is strictly increasing on its support so that there is a one-to-one relation between endowment and rank). There are several advantages of indexing by rank over indexing by endowment level as discussed in detail in Hopkins and Kornienko (2009) and in Section 3 here. Nonetheless, we will use both methods, with indexing by level of endowment to be found in Section 5. In this section, we will treat each competitor's type as her rank  $r$ . Notice that an agent's endowment can be expressed as a function of her rank or  $\tilde{z} = G^{-1}(\tilde{r})$  (i.e.  $\tilde{z}$  is at the  $\tilde{r}$ -quantile). In particular, let us write  $Z(r) = G^{-1}(r)$ . Thus, her strategy will be a mapping  $x(r) : [0, 1] \rightarrow \mathbf{R}_+$  from rank to performance.

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<sup>6</sup>See Cole, Mailath and Postlewaite (1992, 1998) for explicit consideration of voluntary matching between contestants and potential partners.

<sup>7</sup>Nothing substantial depends on this assumption. All results are qualitatively the same if  $x$  has no intrinsic value for contestants.

Then, a symmetric equilibrium will be a Nash equilibrium in which all contestants use the same strategy, that is, the same mapping  $x(r)$  from rank in endowments to performance. Suppose for the moment that all contestants adopt such a strategy  $x(r)$  that, furthermore, is differentiable and strictly increasing (we will go on to show that such an equilibrium exists). Let us aggregate all the performance decisions of the contestants into a distribution function  $F(x)$ . If  $x(r)$  is strictly increasing, then there will be no mass points in the distribution of performance, so that  $F(x)$  is continuous and strictly increasing. Note that when all contestants follow such a strategy, the outcome would be fully separating. One can deduce a contestant's endowments  $z$  or his rank in the distribution of endowments  $r$  from his choice of performance  $x$ .

We assume that formal *equality of opportunity* holds, so that rewards are assigned to contestants solely on the basis of an agent's visible performance,  $x$ .<sup>8</sup> In contrast, *inequality of opportunity* would be exhibited by a rule whereby the allocation of rewards depended on some further, extraneous factor such as race, age, gender or social status.<sup>9</sup>

Given that rewards are indivisible and are ranked from lowest to highest, the obvious way to assign rewards in a way that would satisfy equality of opportunity is assortatively: rewards are assigned on the basis of one's rank  $F(x)$  in performance with the highest performance obtaining the highest reward and so on. This assignment should also be measure-preserving (the equivalent, given a continuum of prizes and contestants, of awarding exactly one prize to each contestant). A possible way to do this is to assign rewards assortatively so that rank in rewards equals rank in endowments, or  $H(s) = G(z)$ . Note that in a symmetric Nash equilibrium where the strategy  $x(r)$  is strictly increasing in an agent's rank, we have that  $G(z) = F(x) = r$ . That is, an agent's rank  $r$  in the distribution of endowments  $G(z)$  is equal to her rank in the distribution of performance. In turn, if rewards are assigned assortatively according to performance so that an agent's rank in the distribution of performance  $F(x)$  is equal to her rank in the distribution of rewards  $H(s)$ , so that  $G(z) = F(x) = H(s) = r$ . Then we have an assignment that satisfies equality of opportunity and is measure preserving.

**Remark 1** *Equality of opportunity implies that rewards are assigned assortatively based on a competitor's performance  $x$  so that the rank of the reward  $H(s)$  is equal his/her rank in the distribution of performance  $F(x)$ . In a fully separating equilibrium, this is equal to his/her rank in endowments so that*

$$G(z) = F(x) = H(s) = r. \tag{2}$$

*That implies that, in such an equilibrium, an agent of rank  $r$  is allocated a reward  $s = H^{-1}(r)$ .*

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<sup>8</sup>Roemer (1996, p. 163) defines formal equality of opportunity as "there is no legal bar to access to education, to all positions and jobs, and that all hiring is meritocratic".

<sup>9</sup>Schotter and Weigelt (1992) consider the effect of inequality of opportunity in stochastic contests with two contestants and find that both theoretically and experimentally that it reduces effort. Similar analysis within our model would call for more advanced methods as inequality of opportunities here would imply an additional dimension of inequality among agents, and thus we leave such analysis for future research.

Note that this relationship (2) implies that we can define the function

$$S(r) = H^{-1}(r), \tag{3}$$

which gives the equilibrium reward of a contestant of type  $r$ , so that  $S : [0, 1] \rightarrow [\underline{s}, \bar{s}]$ . The marginal increase in reward from an increase in one's rank would be

$$S'(r) = \frac{1}{h(H^{-1}(r))} = \frac{1}{h(S(r))}.$$

This also implies a reduced form utility:

$$U(x, y, s) = U(x(r), Z(r) - x(r), S(r))$$

That is, the tournament with assortative award of prizes implies that each individual's payoffs are increasing in her rank  $r$  in the distribution of contestants. It therefore might appear to an outside observer that the individual had some form of social preferences where she cares about her relative position, similar to those analyzed by Frank (1985) and Hopkins and Kornienko (2004). As Cole, Mailath and Postlewaite (1992) as well as Postlewaite (1998) point out, this form of tournament therefore gives a strategic basis to such models.

Continuing with the assumption that all agents adopt the same increasing, differentiable strategy  $x(r)$ , let us see whether any individual agent has an incentive to deviate. Suppose that instead of following the strategy followed by the others, an agent with rank  $r$  chooses  $x_i = x(\tilde{r})$ , that is, she chooses performance as though she had rank  $\tilde{r}$ . Note that her utility would be equal to

$$U = U(x(\tilde{r}), Z(r) - x(\tilde{r}), S(\tilde{r})).$$

We differentiate this with respect to  $\tilde{r}$ . Then, given that in a symmetric equilibrium, the agent uses the equilibrium strategy and so  $\tilde{r} = r$ , this gives the first order condition,

$$x'(r) (U_x(x, Z(r) - x, S(r)) - U_y(x, Z(r) - x, S(r))) + U_s(x, Z(r) - x, S(r))S'(r) = 0. \tag{4}$$

This first order condition balances disutility from increasing effort  $x$  against the implied marginal benefit in terms of an increased reward from doing so. It defines a differential equation,

$$x'(r) = \frac{U_s(x, Z(r) - x, S(r))}{U_y(x, Z(r) - x, S(r)) - U_x(x, Z(r) - x, S(r))} S'(r) = \phi(x, Z(r), S(r)) S'(r). \tag{5}$$

An important point to recognize is that this differential equation and the equilibrium strategy, which is its solution, both depend on the distribution of endowments through  $Z(r) = G^{-1}(r)$  and the distribution of rewards through  $S(r) = H^{-1}(r)$ .

Our next step is to specify what Frank (1985) and Hopkins and Kornienko (2004) call the "cooperative choice", which is the optimal consumption choice  $(x_c(r), y_c(r))$

when an individual cannot affect her reward. Specifically, assume that an agent of rank  $r$  is simply assigned a reward  $S(r)$  rather than having to compete for it. Her optimal choice in these circumstances must satisfy the standard marginal condition

$$U_x(x, Z(r) - x, S(r)) - U_y(x, Z(r) - x, S(r)) = 0. \quad (6)$$

By assumption (v) above, there is a solution  $x_c(r) = \gamma(Z(r), S(r))$  to this maximization problem. The cooperative strategy also enables us to fix the appropriate boundary condition for the differential equation (5). Thus, the initial condition, or the choice of the individual with the lowest rank zero is,

$$x(0) = x_c(0) . \quad (7)$$

We can now show the following existence result. It shows that there is only one fully separating equilibrium. Specifically, if all contestants adopt the strategy  $x(r)$  that is the solution to the above differential equation (5) with boundary condition (7) and rewards are awarded assortatively according to the rule (2), then no contestant has an incentive to deviate. Further, as this solution  $x(r)$  is necessarily strictly increasing, it is fully separating with contestants with high endowments producing a high level of performance. Thus, an authority organizing the tournament to promote equality of opportunity would be rational to give high rewards to high performers as high performance signifies high ability. Or, in the matching story, potential partners should prefer to match with high performers. Note, however, this will typically not be the only equilibrium. As is common in signalling models, other equilibria such as pooling equilibria will exist. In this paper, we concentrate on the separating equilibrium as this seems the most natural for the settings we consider. We now present our result.

**Proposition 1** *The differential equation (5) with boundary conditions (7) has a unique solution which is the only symmetric separating equilibrium of the tournament. Equilibrium performance  $x(r)$  is greater than cooperative amount, that is  $x(r) > x_c(r)$  on  $(0, 1]$ .*

This implies that the cooperative outcome  $x_c(r)$  Pareto dominates the equilibrium performance  $x(r)$  from the point of view of the contestants. As is common in competitive situations, the contestants could make themselves all better off by agreeing to work less. How much more will depend on the exact form of the equilibrium strategy  $x(r)$  which in turn depends on the distribution of endowments and the distribution of rewards. We will go on to look at how equilibrium choices and welfare change in response to changes in these distributions.

Note that this welfare result holds even though contestants derive utility from their own performance, that is, it is not a pure signal. However, if other parties, for example, partners or employers, also benefit from the competitors' efforts, overall welfare judgements are potentially more complicated. Hopkins (2005) looks at this issue and

finds that, in the presence of incomplete information, the level of performance can be excessive even considering the welfare of employers. However, it is clearly true that if contestants' performance is sufficiently valuable to society, then the equilibrium performance level could be excessively low relative to the social optimum even if it too high from their own perspective. Another possibility is that, like in Cole, Mailath and Postlewaite's (1992) original tournament model, the beneficiaries are the next generation. In this case, social competition leads to a growth rate that is higher than the present generation would choose (see also Corneo and Jeanne (1997)).

### 3 Two Effects of Changes in Inequality

In this section we introduce the intuition behind our analysis of how changes in either the distribution of endowments or in the distribution of rewards affect individuals in rank-based tournaments. We make the point that in both cases a change influences individual welfare through two channels, a direct effect and an incentive effect. It is the second effect which is central to our tournament model in that here, in contrast to standard models, changes in the endowment or rewards of others will change the incentives of individuals to engage in effort. But even the direct effect is not straightforward as it can be positive or negative depending on whether it is viewed from a position of a constant endowment or from a constant rank. These differing effects we now try to make clear in a simple way before moving to formal results in the next section.

Consider first a situation where individuals differ in their natural endowments, such as talent, ability, physical attractiveness, and so on. Then, while the distribution of endowments may change, through immigration for example, the endowment of an individual will stay the same. However, if the distribution does change, then typically the rank of such an individual will change even if her endowment does not. In such case, it makes sense to fix an individual by the *level* of her endowment  $z$ , and consider what happens as her *rank*  $r$  changes in response to changes in the distribution.

Consider instead a situation where individual endowments are in terms of income, wealth, capital goods, and so on. In this case, an individual's endowment is not intrinsic and could be changed. For example, a redistributive tax policy could change the endowments of most (if not all) individuals. In such situations, it makes sense to fix an individual by her *rank* in the distribution of endowments  $r$ , but allow for her *endowment*  $z$  to change as the distribution of endowments varies.<sup>10</sup> In essence, this is exactly what policy analysts typically do by analyzing the consequences of redistributive policies for people occupying different ranks - for example, for the median individual or for lower and upper quartiles.

The distinction among rank-indexing and level-indexing is very important for the

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<sup>10</sup>When interventions are rank-preserving (such as with a proportional tax), analysis at a fixed rank is equivalent doing analysis for a given individual before and after the change.

understanding of the effects of changes in inequality. Not only do the two indexing methods require different comparative statics methods, they also differ in how change in inequality is channeled into individual choices and well-being, as we will now see.<sup>11</sup>

In what follows we assume that there is a change in either the distributions of endowments or in the distributions of rewards, but not both. That is, we do not change both distributions at once. We label the initial distribution  $a$  for *ex ante* and the changed distribution  $p$  for *ex post*. We will consider two regimes. In regime  $G$ , we assume that there is no change in the distribution of rewards  $H_a = H_p = H$  but there is a change in the distribution of endowments  $G_a \neq G_p$ . In regime  $H$ , we assume that there is no change in the distribution of endowments, that is  $G_a = G_p = G$ , but the distribution of rewards changes, i.e.  $H_a \neq H_p$ .

We go on to show how, given equality of opportunity, changes in the inequality of endowments and rewards affect different individuals. We distinguish between two different consequences of changes in the level of inequality, which we call the direct effect and the incentive effect.

### 3.1 The Direct Effect

The *direct effect* is what one would obtain under classical assumptions and it simply arises because changes in the social or economic environment of an individual have direct consequences on that individual's choices and well-being - as they will change her endowment  $z$ , or her rank  $r$ , and/or her reward  $s$ . These direct consequences will vary with the indexing method.

To understand the direct effect, suppose rewards were assigned non-competitively by a social planner according to one's rank in the endowment distribution, i.e.  $H(s) = G(z)$ , leading to the "cooperative" choices as set out in Section 2. Notice first that different endowment distributions imply that almost all individuals with fixed rank  $r$  have different endowments in the two societies, i.e.  $Z_a = G_a^{-1}(r) \neq G_p^{-1}(r) = Z_p$  - even though their equilibrium reward  $S = H^{-1}(r)$  does not change (see Figure 1). In contrast, almost all individuals with fixed endowment  $z$  have different ranks in the two societies, i.e.  $r_a = G_a(z) \neq G_p(z) = r_p$ , and thus different equilibrium rewards  $S_a = H^{-1}(G_a(z)) \neq H^{-1}(G_p(z)) = S_p$  (see Figure 2).

An easy way to understand the differences between the two methods of indexing is to compare Figures 1 and 2, which show similar changes in the distribution of endowments. In both cases, the ex post distribution  $G_p$  is less unequal than the original distribution  $G_a$ . As illustrated in Figure 2, for a fixed level of endowment  $z_1$ , in the less unequal distribution of endowments a low ranked agent will have a lower reward. That is, the direct effect of redistribution is *negative* for low-ranked agents under indexing by

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<sup>11</sup>The same issues arise in assignment models. For example, Costrell and Loury (2004) use what we call rank indexing and Suen (2007) uses level indexing and obtain apparently different results.

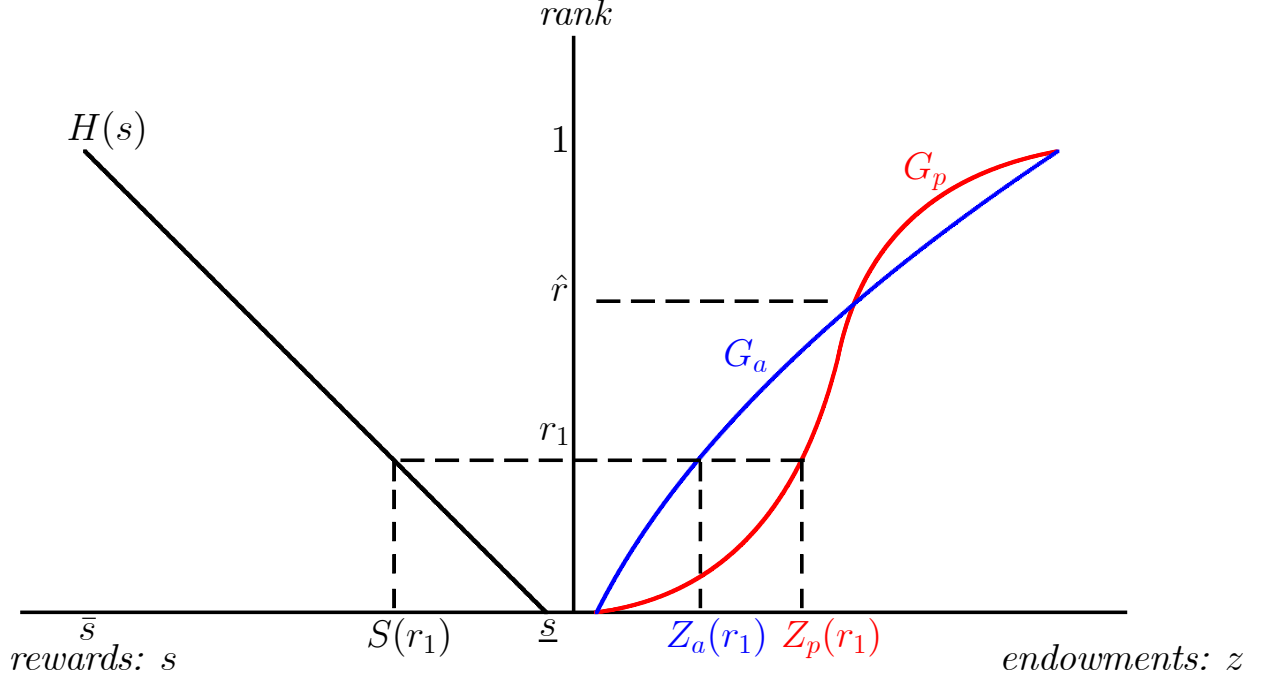


Figure 1: The direct effect in Regime G - under rank-indexing: a contestant with low rank  $r_1$  has a higher endowment  $Z_p(r_1)$  under the less unequal distribution of endowments  $G_p$  than the endowment  $Z_a(r_1)$  under the more unequal distribution of endowments  $G_a$ , and in both cases has a reward  $S(r_1)$ .

endowment levels. However, in Figure 1, it is shown that for a fixed rank  $r_1$  a low ranked agent will have the same reward but a higher level of endowments in a less unequal distribution of rewards, the direct effect of redistribution for the low ranked is *positive*. Comparisons at a fixed level of endowment or at a fixed rank give a very different view of the same phenomenon.

In contrast, when we change the distribution of rewards, the direct effect does not depend on whether we index by rank or by level. The effect of redistribution of rewards will be positive for the low ranked. For example, see Figure 3 where now the ex post distribution of rewards  $H_p$  is less unequal than the ex ante distribution  $H_a(s)$ . We have  $S_a = H_a^{-1}(r_1) = H_a^{-1}(G(z_1)) < H_p^{-1}(r_1) = H_p^{-1}(G(z_1)) = S_p$ . One can also see that it will be negative for the high-ranked.

**Remark 2** *The direct effect of lower inequality can be summarized as follows.*

(i) *Consider first rank-indexing. Suppose endowments become less unequal, then, in equilibrium, low (high) ranking agents have higher (lower) endowments. Suppose, instead, rewards become less unequal, then, in equilibrium, low (high) ranking agents have higher (lower) rewards.*

(ii) *Consider now level-indexing. Suppose endowments become less unequal, then, in*

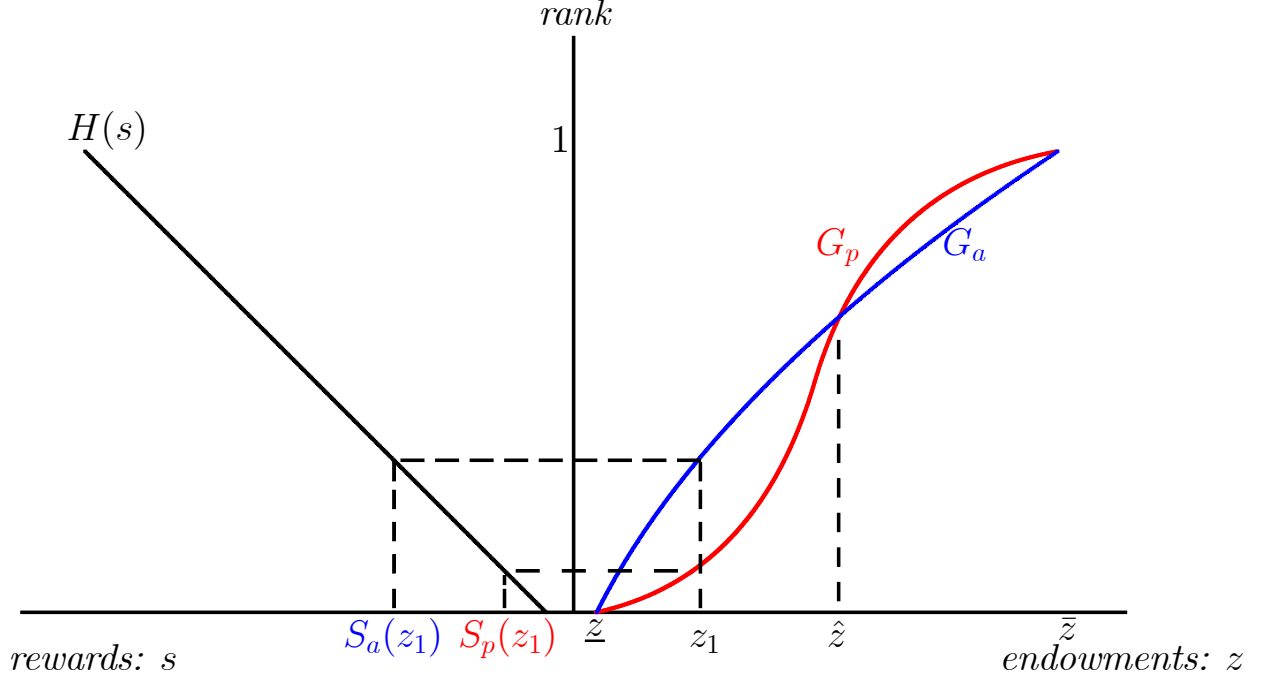


Figure 2: The direct effect in Regime G - under level-indexing: a contestant with fixed low endowment  $z_1$  has a reward  $S_p(z_1)$  under the less unequal distribution of endowments  $G_p$  that is worse than the reward  $S_a(z_1)$  under the more unequal distribution of endowments  $G_a$ .

*equilibrium, low (high) ranking agents have lower (higher) rewards. Suppose, instead, rewards become less unequal, then, in equilibrium, low (high) ranking agents, have higher (lower) rewards.*

Importantly, under rank indexing, the inequality of rewards and endowments appear to have qualitatively similar patterns of benefits and losses when one looks only at the direct (or classical, non-competitive) effect, which may explain why reward and endowment inequality have not been distinguished before. Though, note that under level indexing, the direct effect of lower inequality of endowments is opposite to that of lower inequality of rewards.

### 3.2 The Incentive Effect

Now let us turn to the *incentive* (or marginal, positional, strategic, or social competitiveness) effect of changes in inequality. Importantly, the effect of lower dispersion in rewards and endowments have an opposite incentive effect regardless of the indexing method used. The incentive effect is the result of agents' strategic interactions. As was shown in Hopkins and Kornienko (2004, 2009), in the non-cooperative game where agent's rank matters for her welfare, the "social density", or "social competitiveness",

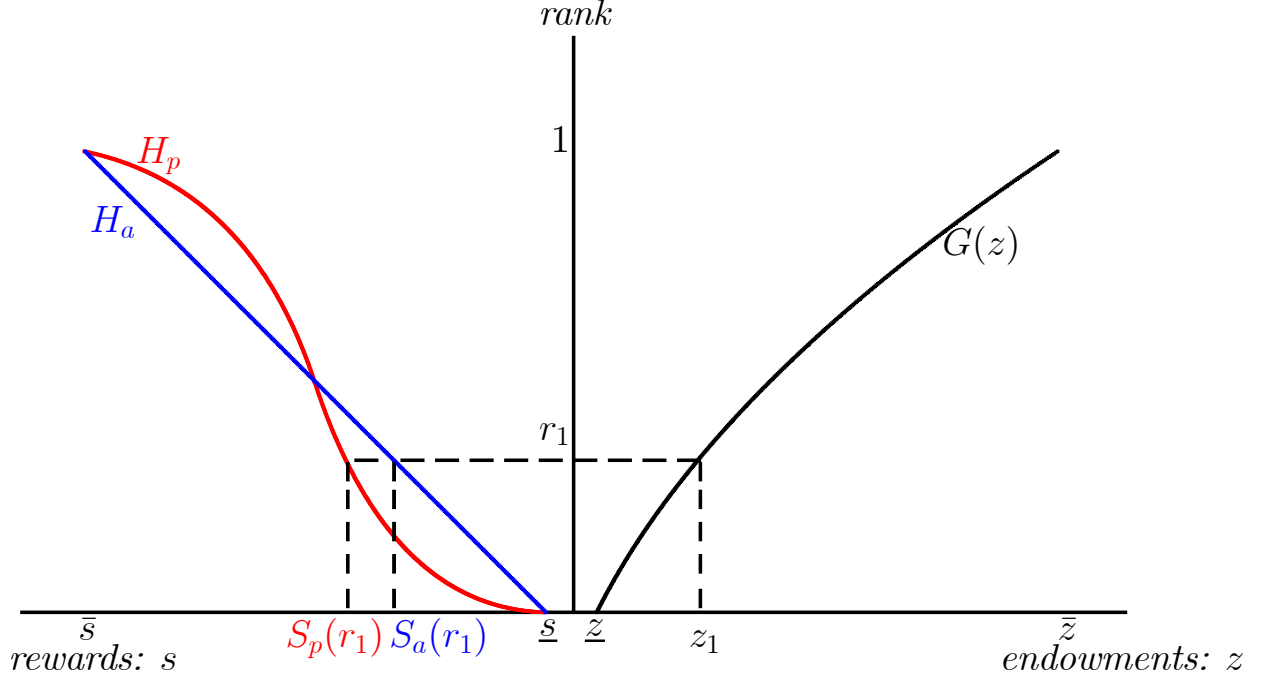


Figure 3: The direct effect in Regime H - under rank- and level-indexing: a contestant with low rank  $r_1$  has higher reward  $S_p(r_1)$  under the less unequal distribution of rewards  $H_p$  than reward  $S_a(r_1)$  under the more unequal distribution of rewards  $S_a$ .

is important as it changes incentives. The incentive effect of changes in distributions on individual choices and welfare will depend largely on the densities of endowments and rewards,  $g(z)$  and  $h(s)$ . This incentive effect can be modeled using the dispersion order (presented in Appendix A) which is a stochastic order used to compare distributions in terms of their densities.

**Remark 3** *The incentive effect of lower inequality can be summarized as follows.*

(i) *Suppose endowments become less dispersed, then there is an increase in the marginal return to effort, as it is now easier to surpass neighbors, so that agents tend to increase their effort.*

(ii) *Suppose rewards become less dispersed, then there is a decrease in the marginal return to effort as rewards are now more similar, so that agents tend to decrease their effort.*

To find the *total effect*, which includes both direct and incentive effects, one needs to analyze how changes in inequality affects behavior, which we turn to now.

## 4 Effects of Changing Inequality Under Indexing By Rank

We will now consider the effect on equilibrium utility and strategies of changes in the distribution of endowments  $G(z)$  and changes in the distribution of rewards  $H(s)$ . In this section, we do this by comparing behavior before and after the change at each rank in the distribution of endowments, using the rank indexing methodology as discussed in Section 3. We saw in Section 2 that equilibrium behavior depends on the reward function  $S$  which is jointly determined by  $G$  and  $H$ . Thus, as the distribution of endowments  $G$  or the distribution of rewards  $H$  change, so does the reward function  $S$ . Thus, a change in either distribution of endowments or rewards (or both) translates into a change in equilibrium choice of performance  $x(r)$  and, thus, into a change in individual welfare.

Equilibrium utility in terms of rank will be  $U(r) = U(x(r), Z(r) - x(r), S(r))$ . By the envelope theorem we have

$$U'(r) = \frac{U_y(x(r), Z(r) - x(r), S(r))}{g(Z(r))} \quad (8)$$

Note that as average utility is  $\int U(r)dr$ , if individual welfare  $U(r)$  rises at every rank then social welfare will surely rise.

In what follows we assume that there is a change in either the distributions of endowments or in the distributions of rewards, but not both. In doing this, we make use of the dispersion order, which as the name suggests, is a way of ordering distributions in terms of their dispersion. Please see Appendix A for details. Our results with respect to inequality of endowments are a generalization of those in Hopkins and Kornienko (2009).

### 4.1 Change in Endowments (Regime G)

We investigate in this section the effects of changes in the distribution of endowments on equilibrium performance decisions and equilibrium utility. In particular, we find that a decrease in the inequality of endowments can have adverse effects. This is because as peoples' endowments become closer together, it is easier to overtake one's neighbors. This leads to a general increase in social competition. While redistribution can benefit those who receive higher endowments, even some of these will be worse off as a consequence of greater competition.

In regime  $G$ , we assume that the societies have identical distributions of rewards, i.e.  $H_a = H_p = H$ , but differ in the distributions of endowments, i.e.  $G_a \neq G_p$  and in fact are distinct, that is, equal at only a finite number of points. Different endowments imply that the two societies have different endowment functions, i.e.  $Z_a = G_a^{-1}(r)$  and  $Z_p = G_p^{-1}(r)$ .

Our first result is to show that if a range of contestants receive an increase in endowments, they will respond with higher performance.

**Proposition 2** *Suppose that endowments are higher ex post so that  $Z_p(r) \geq Z_a(r)$  on an interval  $[0, \hat{r}]$  where  $\hat{r}$  is the point of first crossing of  $Z_p(r)$  and  $Z_a(r)$ . Then  $x_p(0) \geq x_a(0)$  and ex post performance is higher on that interval:  $x_p(r) > x_a(r)$  on  $(0, \hat{r}]$ .*

As a consequence, if the new distribution of endowments  $G_p$  stochastically dominates the old, then performance will be higher for all agents. Note that if  $G_p$  stochastically dominates  $G_a$  then by definition  $G_p(z) \leq G_a(z)$  for all  $z$ , which in turn implies that  $Z_p(r) \geq Z_a(r)$  for all  $r \in [0, 1]$ . That is, in a richer society where endowments are higher for every agent, performance is higher for all.

**Corollary 1** *Suppose that endowments are stochastically higher ex post so that  $Z_p(r) \geq Z_a(r)$  for all  $r \in [0, 1]$ , then performance rises almost everywhere:  $x_p(r) > x_a(r)$  on  $(0, 1]$ .*

We can now give a sufficient condition for equilibrium utility to rise for all agents and hence for an increase in social welfare. The condition has two parts. First, endowments must be more dispersed in the sense of the dispersion order, or  $G_p \geq_d G_a$  (see Appendix A for the definition and properties of this and subsequently used stochastic orders). Second, the lowest ranked agent must be no worse off, or  $Z_p(0) \geq Z_a(0)$ . The point is that, as utility both depends on endowments and the degree of social competition, one can guarantee an increase in endowments will lead to an increase in utility if at the same time the social density does not rise.

**Proposition 3** *Suppose endowments are more dispersed ex post  $G_p \geq_d G_a$  and minimum endowments no lower  $Z_p(0) \geq Z_a(0)$ , then utility is higher ex post almost everywhere:  $U_p(r) > U_a(r)$  on  $(0, 1]$ .*

Our final result in this subsection concerns a decrease in inequality. As remarked, there are two resulting effects. Figure 1 illustrates the direct effect: with a less unequal distribution of endowments, the low ranked have higher endowments ex post. However, as we have argued, the marginal effect works toward greater competition. As people are closer together, it is easier to overtake one's neighbors. For the low ranked, the direct effect dominates. For the middle class, the marginal effect is more important, whereas for the upper classes, they lose both from redistribution and from greater competition. We thus find that the middle and upper classes are worse off. This is illustrated in Figure 4.

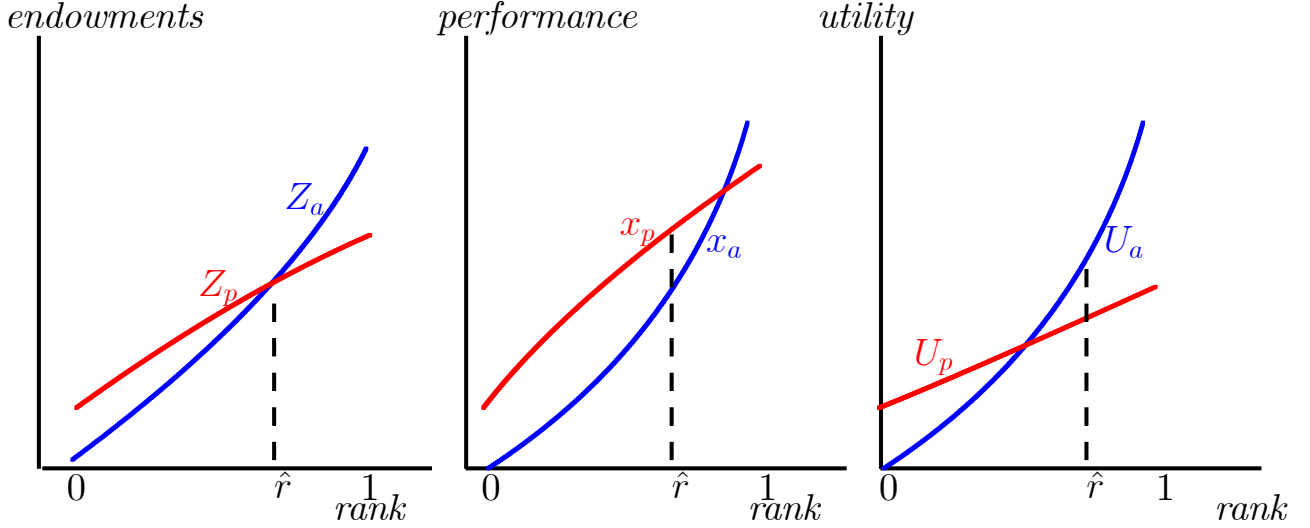


Figure 4: Less unequal endowments, indexing by rank: typical comparative statics when ex post endowments  $Z_p$  are less unequal than ex ante  $Z_a$ . Performance rises at lower and middle ranks; but utility falls at middle and upper ranks.

Specifically, we suppose the distribution of endowments becomes less dispersed in terms of the dispersion order. Furthermore, the lowest ranked agent has more endowments, or  $Z_p(0) > Z_a(0)$ , and the highest ranked has less  $Z_p(1) < Z_a(1)$ . Thus, in a clear sense the distribution  $G_p$  of endowments is less unequal than distribution  $G_a$ .

**Proposition 4** *Suppose that the minimum level of endowments is higher ex post*

$$Z_p(0) > Z_a(0) \quad (9)$$

*and endowments are less dispersed ex post*

$$g_p(Z_p(r)) \geq g_a(Z_a(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_a \geq_d G_p \quad (10)$$

*and also suppose that the maximum level of endowments is lower ex post*

$$Z_p(1) < Z_a(1) \quad (11)$$

*Then, performance is higher ex post for the bottom and middle:  $x_p(r) > x_a(r)$  on  $[0, \hat{r}]$  where  $\hat{r}$  is the only point of crossing of  $Z_a(r)$  and  $Z_p(r)$ . Second, utility rises at the bottom,  $U_p(0) > U_a(0)$ , but utility is lower ex post for the middle and top,  $U_p(r) < U_a(r)$  for all  $r \in [\hat{r}, 1]$ .*

Note that this result implies that there are middle ranking agents who are worse off even though they have higher endowments ex post (again see Figure 4 for the outcomes for individuals just to the left of  $\hat{r}$ ). However, the effect at the relatively low ranked individuals, i.e. those with  $r \in (0, \hat{r})$  is, in general, ambiguous.

## 4.2 Changes in Rewards (Regime H)

In this subsection, we find that the effects of changes in rewards are quite different from those arising from changes in endowments. The first point is that the effect of a decrease in inequality in rewards has the opposite incentive effect to a decrease in inequality of endowments. Lower inequality of rewards implies that the marginal return to greater effort is relatively low, and will tend to reduce competition. This will tend to make competitors better off. However, for high ranking competitors who expect high rewards, the effect is ambiguous. In a less unequal society they work less hard but obtain lower rewards.

In regime  $H$ , we assume that the societies have identical distributions of endowments, i.e.  $G_a = G_p = G$ , but differ in the distributions of rewards, i.e.  $H_a \neq H_p$  and in fact are distinct, that is, equal at only a finite number of points.. Again, different rewards imply that the two societies have also different reward functions, i.e.  $S_a(r) = H_a^{-1}(r)$  and  $S_p(r) = H_p^{-1}(r)$ .

Our first result concerns sufficient conditions for greater effort by all competitors. We find that if rewards are lower at every rank and the rewards are more dispersed, then the environment is definitely more competitive and effort rises at every rank.

**Proposition 5** *Suppose that the rewards are more dispersed ex post*

$$S'_p(r) \geq S'_a(r) \text{ on } (0, 1) \Leftrightarrow h_p(S_p(r)) \leq h_a(S_a(r)) \text{ on } (0, 1) \Leftrightarrow H_p \geq_d H_a \quad (12)$$

*and that the minimum reward is lower ex post*

$$S_p(0) < S_a(0) \quad (13)$$

*and then performance is higher ex post so that  $x_p(r) > x_a(r)$  on  $(0, \hat{r}]$  where  $\hat{r}$  is the first crossing point of  $S_p(r)$  and  $S_a(r)$ .*

This leads to the following corollary. If rewards are more unequal and lower at every rank, then performance increases for every agent.

**Corollary 2** *Suppose that the ex-post rewards are more dispersed and also are stochastically lower, i.e.  $H_p \geq_d H_a$  and  $S_p(r) \leq S_a(r)$  for all  $r \in [0, 1]$ , then performance rises almost everywhere:  $x_p(r) > x_a(r)$  on  $(0, 1]$ .*

Note that if one makes stronger assumptions on the utility function, one can still obtain an increase in performance at all ranks without the stochastic dominance assumption of Corollary 2. First, we look at the case of utility being additively separable in rewards.

**Proposition 6** *Assume utility is additively separable in rewards, that is  $U = V(x, y) + s$  for some function  $V$  such that conditions (i) to (v) on  $U$  are still satisfied, then if  $H_p \geq_d H_a$ , it follows that  $x_p(r) > x_a(r)$  almost everywhere on  $[0, 1]$ .*

We can obtain a similar result if utility is multiplicatively separable in rewards. For such preference specification, we will use another method of comparing distributions, the star order. This order is defined and discussed in detail in Appendix A. Informally, the star order implies that  $H_p$  is more dispersed or stochastically lower than  $H_a$  but not necessarily both. Compare the next result to Corollary 2), where we assumed that  $H_p$  is both more dispersed and stochastically lower than  $H_a$ .

**Proposition 7** *If rewards are multiplicatively separable or  $U = V(x, y)s$  for some function  $V$  such that conditions (i) to (v) on  $U$  are still satisfied, then if  $H_p$  is more dispersed in the star order than  $H_a$ , or  $H_p \geq_* H_a$ , it follows that  $x_p(r) > x_a(r)$  almost everywhere on  $[0, 1]$ .*

We next identify a sufficient condition for an increase in equilibrium utility at every rank. This is much simpler than when considering changes in the distribution of endowments. Here, we simply require that the new distribution  $H_p$  stochastically dominates the old  $H_a$  and that the lowest reward  $S_p(0)$  is strictly higher. This implies that  $S_p(r) \geq S_a(r)$  for all  $r$ , or rewards are higher at every rank. As this will also decrease the incentives to compete, it is not surprising that equilibrium utility rises.

**Proposition 8** *If the minimum reward is higher ex post  $S_p(0) > S_a(0)$  and rewards are everywhere else no lower,  $S_p(r) \geq S_a(r)$  for all  $r \in (0, 1]$ , then utility is everywhere higher ex post:  $U_p(r) > U_a(r)$  on  $[0, 1]$ .*

We now turn to inequality. As illustrated in Figure 3, the direct effect of lower inequality in rewards benefits the low-ranked simply because their rewards will typically be higher. Furthermore, the compression of rewards will decrease the marginal incentive to compete and performance will fall. Thus, this marginal effect will further benefit contestants. Thus, as we see in Figure 5, utility will rise even for the agent with rank  $\hat{r}$  whose reward is unchanged.

**Proposition 9** *Suppose that the lowest reward is higher ex post*

$$S_p(0) > S_a(0) \tag{14}$$

*and also rewards are less dispersed ex post*

$$S'_p(r) \leq S'_a(r) \text{ for all } r \in (0, 1) \Leftrightarrow H_a \geq_d H_p \tag{15}$$

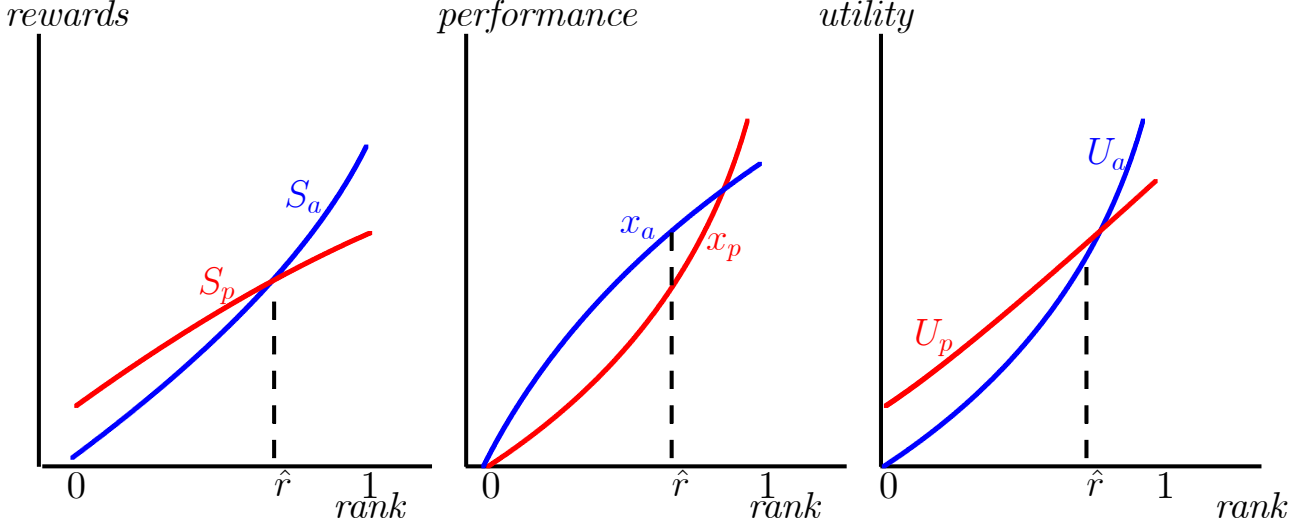


Figure 5: Less unequal rewards, indexing by rank: typical comparative statics when ex post rewards  $S_p$  are less unequal than ex ante  $S_a$ . Performance falls and utility rises at low and middle ranks.

and also suppose that the highest reward is lower ex post

$$S_p(1) < S_a(1). \quad (16)$$

Then performance is lower ex post  $x_p(r) < x_a(r)$  on  $(0, \hat{r}]$  where  $\hat{r}$  is the only point of crossing of  $S_a(r)$  and  $S_p(r)$ . Second, utility is higher on that interval:  $U_p(r) > U_a(r)$  for all  $r \in [0, \hat{r}]$ .

We have already seen, Propositions 6 and 7, that in some special cases, a reduction in the dispersion of rewards is sufficient to make performance fall for all competitors. We give an example of this, which has another interesting property.

**Example 1** Assume that  $U(x, y, s) = x^\alpha y s$  for some  $\alpha < 1$ , so rewards are multiplicatively separable, and that endowments are uniform on  $[1, 2]$ . Suppose, for example, rewards go from being uniform on  $[0.5, 2.5]$  ( $H_a = 0.5s - 0.25$  or  $S_a = 2r + 0.5$ ) to being uniform on  $[1, 2]$  ( $H_p = s - 1$  or  $S_p = r + 1$ ). Then, by Proposition 7, performance must fall almost everywhere as these two distributions satisfy  $H_p \leq_* H_a$ , the ex post distribution is less dispersed in terms of the star order (and, also, the dispersion order). Note that the lowest competitor would have a higher utility under the ex post distribution, i.e.  $U_p(0) > U_a(0)$ , as she has a higher reward (but the same endowment). Indeed, everyone with rank up to 0.5 must be better off by Proposition 9 as here the crossing point of  $S_a$  and  $S_p$  is 0.5. But, further, here  $U'(r) = x^\alpha(r) Z'(r) S(r)$ . If  $\alpha$  is reasonably low so that the influence of the lower performance ex post is not large, the slope of utility in rank will not be very different ex post. Thus, for example, for  $\alpha < 0.35$ , everyone will be better off under the less dispersed distribution  $H_p$ .

That is, by making rewards less dispersed, it is possible to reduce total performance but make a Pareto improvement. Everyone will be happier because everyone works less. This raises the question as to whether it would be possible to make everyone better off by altering the level of inequality of endowments. However, while a greater dispersion of endowments by Proposition 4 reduces performance for most (and possibly all) competitors, it cannot make all better off for a fixed average endowment. This is because the greater dispersion would lower the utility of low ranked competitors, as they would have lower endowments in the more dispersed distribution.

## 5 Results under Indexing by Level of Endowment

We now consider a situation where the endowment is intrinsic to the agent, for example, talent. We, therefore, use the level-indexing method and compare an agent's utility before and after changes in the level of inequality given this fixed level of endowment. As this method has been used before, for example by Hopkins and Kornienko (2004) and Hopkins (2005), it thus requires less extensive coverage. We find an apparently different outcome from that under rank indexing as those with low endowments are now worse off under lower inequality of endowments. The reason for this is that, as discussed in Section 3, the direct effect of lower inequality on an individual on a fixed low level of endowments is negative, as opposed to positive under rank indexing.

We now look at the tournament from the perspective of indexing by levels of endowments. That is, we consider the model introduced in Section 2 in terms of endowments  $z$  not rank  $r$ . As before a continuum of contestants choose  $x$  to maximize utility (1). Given the assortative assignment of rewards (2), we can now write the equilibrium reward as a function of endowment as  $S(z) = H^{-1}(G(z))$ . We look for a strictly increasing symmetric equilibrium strategy as a function of endowments. The equilibrium strategy  $x(z)$  will be a solution to the following differential equation, compare equation (5),

$$\frac{dx(z)}{dz} = \frac{U_s(x, z - x, S(z))g(z)}{U_y(x, z - x, S(z)) - U_x(x, z - x, S(z))h(S(z))} = \frac{dx(r)}{dr} \frac{dr}{dz} = \frac{dx(r)}{dr} g(z). \quad (17)$$

The boundary condition will be  $x(\underline{z}) = x_c(G(\underline{z}))$ , that is the same as in rank terms (7). The only separating equilibrium in terms of endowments  $x(z)$  will be a solution to the above equation. This is a direct consequence of Proposition 1. Working in terms of endowments or ranks does not change the underlying game or its equilibria. We emphasize that they are just different ways of looking at the same behavior.

We will also look at individual welfare in terms of endowments. Define  $U(z) = U(x(z), z - x(z), S(z))$ , that is  $U(z)$  is equilibrium utility in terms of endowments  $z$ . We show that a decrease in inequality of endowments amongst competitors reduces the utility of the weakest competitors. In contrast, a similar decrease in the dispersion of the rewards has an opposite effect. In contrast to our work using rank-indexing, we assume here that  $G_a$  and  $G_p$  have the same support  $[\underline{z}, \bar{z}]$  and that similarly there is a

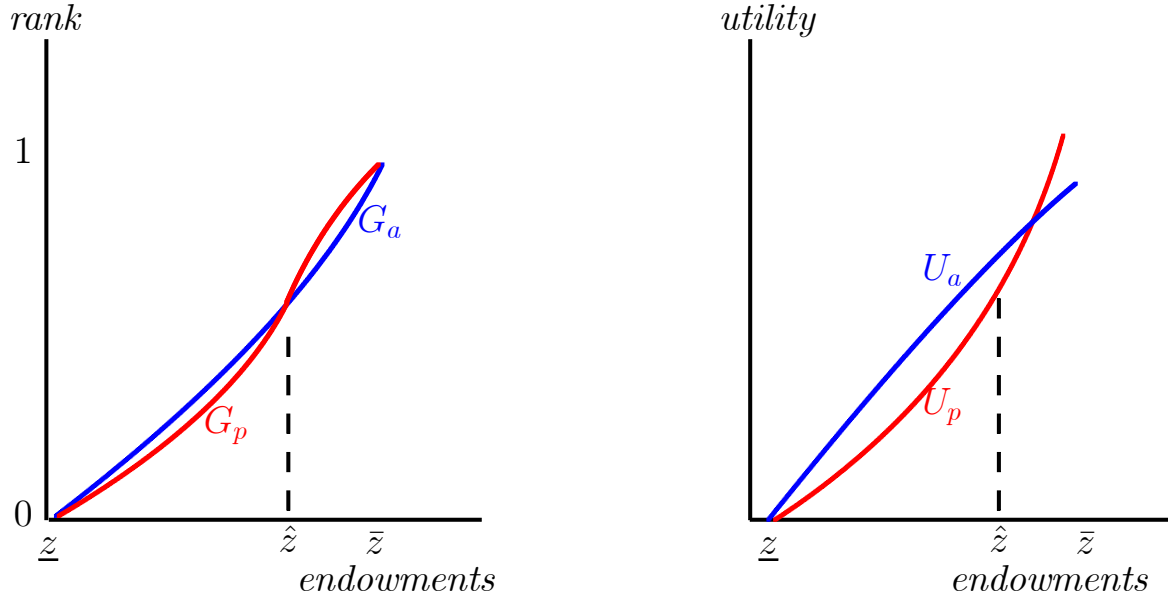


Figure 6: Less unequal endowments, indexing by levels: typical comparative statics when the ex post distribution of endowments  $G_p$  is less unequal than ex ante  $G_a$ . Utility falls at low and middle levels of endowments.

common support  $[\underline{s}, \bar{s}]$  for the distributions of rewards  $H_a$  and  $H_p$ . Here we use second order stochastic dominance to order distributions in terms of dispersion (see Appendix A for the relationship among different stochastic orders).

**Proposition 10** *Let  $U_a(z)$  and  $U_p(z)$  be the equilibrium utilities in terms of endowments ex ante and ex post respectively.*

(i) *Suppose that  $G_p$  second order stochastically dominates  $G_a$ . Denote the first crossing of  $G_a(z)$  and  $G_p(z)$  as  $\hat{z}$ . Then, utility falls for the bottom and middle  $U_p(z) \leq U_a(z)$  for all  $z \in [\underline{z}, \hat{z}]$ .*

(ii) *Suppose that  $H_p$  second order stochastically dominates  $H_a$ . Denote the first crossing of  $H_a(s)$  and  $H_p(s)$  as  $\hat{s}$ , and denote  $\hat{z} = S^{-1}(\hat{s}) = G^{-1}(H_p(\hat{s})) = G^{-1}(H_a(\hat{s}))$ . Then, utility rises for the bottom and middle  $U_p(z) \geq U_a(z)$  for all  $z \in [\underline{z}, \hat{z}]$ .*

That is, for those whose endowments are relatively low (e.g. less than  $\hat{z}$  in Figure 6), a less unequal distribution of endowments leads to lower individual welfare, while, conversely, a similar decrease in inequality of rewards results in an increase in individual welfare. This is because, as discussed in Section 3, for an individual with a fixed low level of endowment, the direct effect of lower inequality is negative, in that she will now have a lower reward (again see Figure 2). This is because with the reduction in inequality there are more contestants with middling endowments who will now take the middling rewards. The incentive to compete is also increased by the greater social density and so even those in the middle will be worse off as they compete harder. Conversely, the direct effect of less unequal rewards is positive and incentives to compete are reduced. Thus, the results for reduced inequality of rewards for level indexing are qualitatively similar to those for rank indexing depicted in Figure 5.

## 6 Discussion and Conclusions

This paper introduces a new distinction between different kinds of inequality. Inequality of initial endowments and inequality of the final rewards to success in society have opposing effects. Greater inequality of endowments decreases the degree of social competition, greater inequality of rewards increases it. Thus, it is not the case that greater inequality necessarily decreases happiness. Rather, it is inequality of rewards, not of endowments, that is a likely cause of concern.

There has been much recent work concerned with the possibility that people have intrinsic preferences over the level of inequality. Here, we offer a reason why inequality may matter even without any concern for social justice and in the absence of such social preferences. This is because when there is interpersonal competition for employment and educational opportunities, inequality has a direct impact on incentives and, hence, equilibrium effort and equilibrium utility. The competitive threat of being excluded from desirable opportunities means that, in equilibrium, everyone works too hard. This means that people can be made better off by a change in incentives implicit in the two different forms of inequality. The majority can gain from a more dispersed distribution of endowments or from a less dispersed distribution of rewards. In fact, we can construct examples where a reduction in the inequality of rewards makes everyone better off, that is, it is Pareto improving, even though this reduction in incentives decreases total performance.

It is true that if contestants' efforts benefit other agents, such as partners, employers or members of future generations, then there is a stronger case for reward inequality. However, there remains a question as to whether those who lose from such inequality are ever compensated. For example, gains to future generations may not be sufficient recompense to those who lose now from greater inequality of rewards. Or, as another possibility, societies with high inequality of rewards may have higher growth but lower happiness for a given level of per capita income than societies with lesser inequality of rewards. Thus, one clear direction for further research is to use the current model as the stage game in a dynamic setting. Preliminary results in this direction indicate that the effects of changes in inequality on growth depend heavily on whether current performance determines the rewards or the endowments of the next generation.

As we demonstrated in this paper, the relationship between inequality and individual welfare can be less straightforward than is commonly thought. The gains and losses to greater inequality even differ according to the viewpoint taken, that is, whether we compare at a constant level of endowment or at a constant rank in society. However, rather than being a setback, we believe the richness of the relationships we have outlined and the tools we have developed to analyze them offer many possibilities for greater understanding of social phenomena.

For example, one of the more recent reasons advanced for the desirability of greater income equality is the presence of relative concerns. It has been argued that in countries

where gross poverty has been eliminated, health tends to be driven by stress caused by one's relative position, which, in turn, is exacerbated by inequalities. The most famous single case study is that of British civil servants, where health was found to be very strongly positively correlated with a civil servant's rank in the service (Marmot et al. (1991)). It has been argued by several authors, notably Frank (1999, 2000), that if utility does depend on relative position, greater equality should be socially beneficial. However, Deaton (2003) argues that the empirical evidence as a whole does not support a general link between inequality and ill health. Furthermore, it has been difficult to establish whether there is a positive or negative relationship between inequality and self-reported happiness or life-satisfaction (Alesina et al. (2004), Clark (2003)).

This paper suggests a reason why this may be the case. Even when utility depends on relative position, different types of inequality may have opposite effects. Therefore, empirical work that is based on measures of inequality that conflate rewards and endowments may obtain weak results as the two opposing effects may cancel. The problem in immediately applying this insight to empirical problems is that, to our knowledge, no distinction between reward and endowment inequality has traditionally been made in data collection. However, with data sources such as longitudinal studies becoming more widely available, it may soon be possible to distinguish between initial endowments and final rewards.

Finally, we would like to emphasize that, although this work approaches inequality outside the framework of distributive justice, it does not mean that moral considerations are irrelevant to the issue of inequality. In fact, precisely because existing theories of justice do not give interpersonal competition such a central role, our tournament model may provide new tools and new insights that may be useful to researchers on distributive justice. Thus, we hope that this paper, even though it takes a purely economic approach, may aid our understanding of inequality in many of its aspects.

## Appendix A: The Dispersive, Star and Other Stochastic Orders

We use two different stochastic orders, the dispersive and the star orders. These may not be well known in economics (though see Hoppe et al. (2009)), but are extremely useful for the social contests we consider. Let  $F$  and  $G$  be two arbitrary continuous distribution functions each with support on an interval (but the two intervals need not be identical or even overlap) and let  $F^{-1}$  and  $G^{-1}$  be the corresponding left-continuous inverses (so that  $F^{-1}(r) = \inf\{x : F(x) \geq r\}$ ,  $r \in [0, 1]$  and  $G^{-1}(r) = \inf\{x : G(x) \geq r\}$ ,  $r \in [0, 1]$ ), and let  $f$  and  $g$  be the respective densities.

**Definition 1** (*Shaked and Shanthikumar (2007, p148)*) *A variable with distribution  $F$  is said to be smaller in the dispersive order (or less dispersed) than a variable with a*

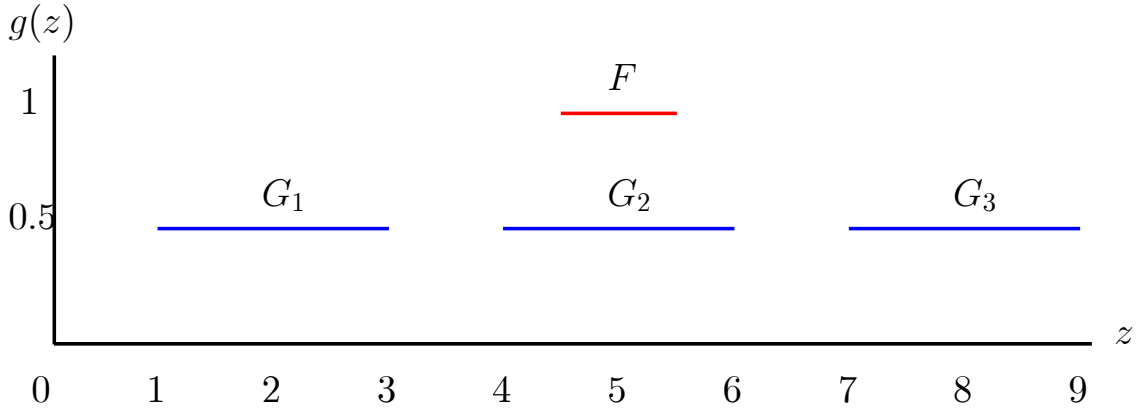


Figure 7: An example of the dispersion order:  $F \leq_d G_1 \sim_d G_2 \sim_d G_3$

distribution  $G$  (denoted as  $F \leq_d G$ ) whenever  $G^{-1}(r) - F^{-1}(r)$  is (weakly) increasing for  $r \in (0, 1)$ .

That is, the difference in the two variables at a given rank increases in rank. This has the following important consequence,

$$G \geq_d F \text{ if and only if } f(F^{-1}(r)) \geq g(G^{-1}(r)) \text{ for all } r \in (0, 1) \quad (18)$$

That is, for a fixed rank, the more dispersed distribution is less dense than the less dispersed one. Note that because the condition (18) is expressed in terms of ranks, there is no problem in comparing distributions with different, even non-overlapping, supports. Finally, when both distributions have finite means, if  $F$  is less dispersed than  $G$  then  $\text{Var}_F(z) \leq \text{Var}_G(z)$  whenever  $\text{Var}_G(z) < \infty$ . Figure 7 shows a simple example of distributions which are ordered in terms of the dispersion order. The distributions  $G_B^1, G_B^2, G_B^3$  all have different means but are equally dispersed and all are more dispersed than  $G_A$ . Figure 8 shows the importance of the dispersion order for incentives in the tournament model: if a distribution  $H_a$  is more dispersed than a distribution  $H_p$  then by (18) necessarily the inverse function  $S_a(r)$  is steeper than  $S_p(r)$ . This is because if  $S(r) = H^{-1}(r)$ , then  $S'(r) = 1/h(H^{-1}(r))$ .

The star order is defined as follows.

**Definition 2** (Shaked and Shanthikumar (2007, p214)). A variable with a distribution  $F$  is smaller in the star order than a variable with a distribution  $G$ , or  $F \leq_* G$ , if  $G^{-1}(F(z))/z$  increases for  $z \geq 0$ .

Note that for two non-negative random variables  $X$  and  $Y$ , the star and dispersive order have the following relationship:

$$X \leq_* Y \iff \log X \leq_d \log Y \quad (19)$$

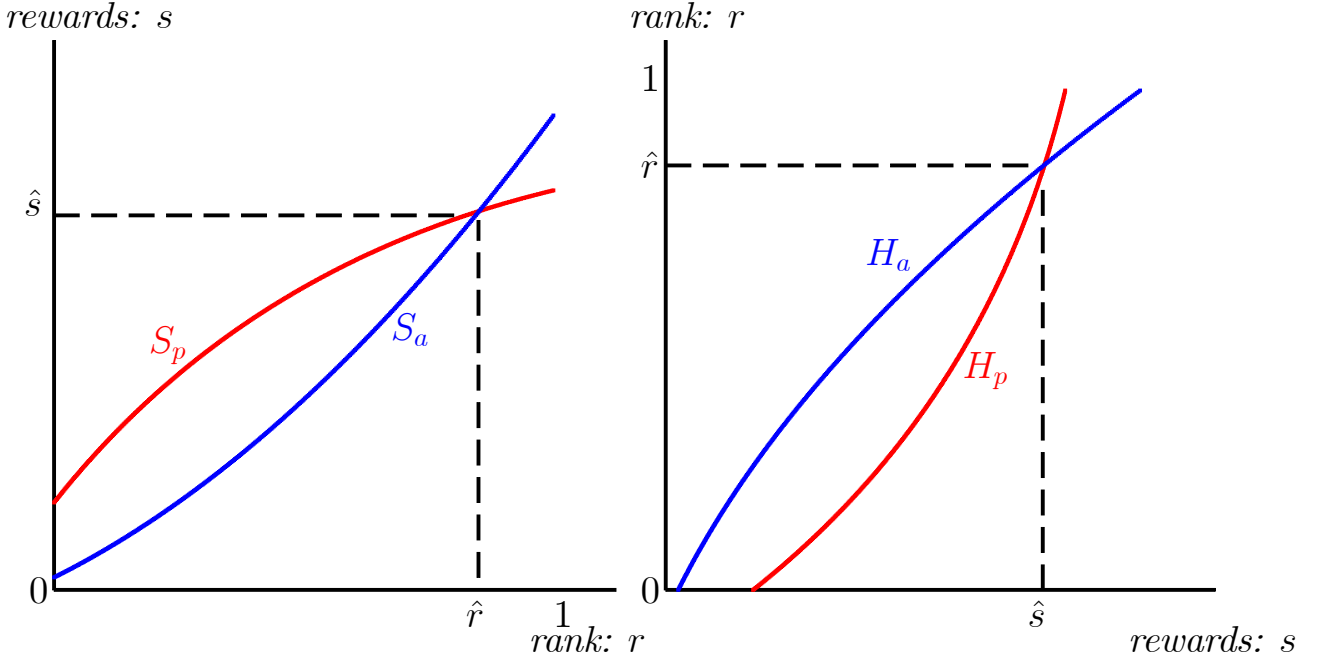


Figure 8: Dispersion order: If the ex post distribution is less dispersed than the ex ante, or  $H_p \leq H_a$  then the inverse distribution function  $S_p = H_p^{-1}(r)$  is less steep than  $S_a$  for all  $r \in (0, 1)$ , i.e. the marginal return to an increase in rank is lower.

However, if a distribution  $F$  is more dispersed than another distribution  $G$ , or  $F \geq_d G$ , it does not imply that  $F$  is larger in the star order,  $F \geq_* G$ , though it is not excluded. Nor does  $F \geq_* G$  imply  $F \geq_d G$ , nor does it rule it out.

**Lemma 1** Take two distributions  $H_a(s)$ ,  $H_p(s)$  with support on the positive real line and with differentiable inverses  $S_a(r)$  and  $S_p(r)$  respectively. Then, the following holds

$$H_p(s) \geq_* H_a(s) \Leftrightarrow \frac{d S_p(r)}{dr S_a(r)} \geq 0 \Leftrightarrow \frac{S'_p(r)}{S_p(r)} \geq \frac{S'_a(r)}{S_a(r)} \quad (20)$$

for all  $r \in (0, 1)$ .

**Proof:** The relationship between the first and second statements follows directly from Shaked and Shanthikumar (2007, p216 and Theorem 4.B.5). The relation between the second and third follows from differentiation. ■

Economists often use second order stochastic dominance to order distributions in terms of dispersion. However, there is no clear relation between the dispersive order and second order stochastic dominance. This is because the second order stochastic dominance relationship compares distributions both on the basis of the mean and dispersion (variance). In contrast, the dispersive order is only concerned with dispersion. For example, in Figure 7, distribution  $F$  second order stochastically dominates distributions  $G_1$  and  $G_2$ , but it is second order stochastically dominated by distribution  $G_3$ .

On the other hand, since the star order implies single crossing of the distribution functions, for two distributions with the same mean we have that if distribution  $H_a$  is larger in the star order  $H_a \geq_* H_p$ , then  $H_a$  second order stochastically dominates  $H_p$  (Shaked and Shanthikumar, 2007, p223, link the star and Lorenz order which is essentially the same as second order stochastic dominance when comparing distributions with the same mean). The following examples demonstrate the relationship between the dispersion order, the star order, and the second order stochastic dominance order.

**Example 2** *If  $H_a(s) = s$ , that it is uniform on  $[0, 1]$  and  $H_p(s) = 2s - 0.5$ , a uniform distribution on  $[0.25, 0.75]$ , then  $H_a$  is more dispersed than  $H_p$ . Indeed,  $S_a(r)/S_p(r) = r/(0.5r + 0.25)$  which is increasing so  $H_a \geq_* H_p$ . Furthermore,  $S'_a(r) = 1 > 0.5 = S'_p(r)$  so that  $H_a \geq_d H_p$ . And finally  $H_p$  second order stochastically dominates  $H_a$ .*

This example illustrates a more substantive difference.

**Example 3** *If  $H_a(s) = s - 2$ , that it is uniform on  $[2, 3]$  and  $H_p(s) = (s - 1)/2$ , a uniform distribution on  $[1, 3]$ , then  $H_p$  is more dispersed than  $H_a$  but stochastically lower. The dispersive order captures the dispersion so as  $S'_a(r) = 1 < 2 = S'_p(r)$  so that  $H_p \geq_d H_a$ . But,  $S_p(r)/S_a(r) = (2r + 1)/(2 + r)$  which is increasing so  $H_p \geq_* H_a$ . However, as  $H_a$  stochastically dominates  $H_p$ , it also second order stochastically dominates  $H_p$ .*

## Appendix B: Proofs

**Proof of Proposition 1:** Mailath (1987) establishes in a general signaling model the existence and uniqueness of a separating equilibrium under certain conditions. If the current model fits within Mailath's framework, then it would follow that the unique separating equilibrium is a solution to the differential equation (5) with boundary condition  $x(0) = x_c(0)$  from Theorems 1 and 2 of Mailath (1987, p1353). It would also follow by Proposition 3 of Mailath (1987, p1362) that  $x(z) > x_c(z)$  on  $(\underline{z}, \bar{z})$ . The only substantial difference is that Mailath assumes the signaller's utility is of the form (in current notation)  $V(r, \hat{r}, x)$  where  $V$  is a smooth utility function and  $\hat{r}$  is the perceived type, so that in a separating equilibrium the signaller has utility  $V(r, r, x)$ . To apply this here, first, fix  $G(z)$  and  $H(s)$ . Now, clearly, one can define the function  $V(\cdot)$  such that  $V(r, \hat{r}, x) = U(x, Z(r) - x, S(\hat{r}))$  everywhere on  $[0, 1] \times [0, 1] \times [\underline{z}, \bar{z}]$ . One can then verify that the conditions (i)-(v) imposed on  $U(\cdot)$  imply conditions (1)-(5) of Mailath (1987, p1352) on  $V$ .<sup>12</sup> In particular, note that condition (1) is simply that  $V$  is twice differentiable, condition (2) is that  $V_2 \neq 0$ , here  $V_2 = U_s S'(r) > 0$ . Condition (3) is

<sup>12</sup>Mailath, in proving the intermediate result Proposition 5 (1987, p1364), also assumes that  $\partial V/\partial \hat{r}$  is bounded. Here, if we assume that both  $U_s$  and  $S'(r)$  are bounded (the latter requires  $h(s)$  is non-zero on its support), this result will also hold.

that  $V_{13} \neq 0$  and here  $V_{13} = (U_{xy} - U_{yy})Z'(r) > 0$ . Mailath's condition (4) requires that  $V_3(r, r, x) = 0$  has a unique solution in  $x$  which maximizes  $V(r, r, x)$ . Here,  $V_3 = U_x - U_y$  and we have assumed under condition (v) that there is a unique solution to the equation  $U_x - U_y = 0$ . Since here  $V_{33} = U_{xx} - 2U_{xy} + U_{yy} < 0$ , this solution is a maximum. Furthermore, since  $V_{33}$  is everywhere negative, Mailath's condition (5) is automatically satisfied. ■

**Proof of Proposition 2:** First note that, given the equation (5), we have that

$$\frac{x'_a(r)}{x'_p(r)} = \frac{\phi(Z_a(r), S(r), x_a)}{\phi(Z_p(r), S(r), x_p)} \quad (21)$$

so that any point where  $x_a = x_p$  the relative slope only depends on  $Z_a$  and  $Z_p$ , and thus the slopes are equal whenever  $Z_a$  and  $Z_p$  are equal. Furthermore, given our assumptions, we have that

$$\frac{\partial\phi(z, s, x)}{\partial z} = \frac{U_{ys}(U_y - U_x) - U_s(U_{yy} - U_{xy})}{(U_y - U_x)^2} > 0 \quad (22)$$

(by properties (iii) and (iv), it holds that  $U_y - U_x > 0$  when evaluated at the equilibrium solution as  $x(r) > x_c(r)$ ). Thus, at any point where  $x_a(r) = x_p(r)$  we have that  $x'_a > x'_p$  (so that  $x_a$  is steeper than  $x_p$  and thus crosses  $x_p$  from below) whenever  $Z_a(r) > Z_p(r)$  (i.e. whenever ex-ante endowments exceed ex-post endowments), and vice versa.

By the boundary condition (7), the condition  $Z_a(0) \leq Z_p(0)$  implies that  $x_p(0) \geq x_a(0)$  (i.e. that the poorest individual, now that she has a greater endowment, chooses greater performance). Given our assumption that  $G_a$  and  $G_p$  are distinct, it follows that  $Z_p(r) > Z_a(r)$  almost everywhere on  $(0, \hat{r}]$ . Thus,  $x_p(r)$  can only cross  $x_a(r)$  from below except perhaps at the finite number of points where  $Z_p(r) = Z_a(r)$ .

We first rule out that there is an interval where  $x_p(r) \leq x_a(r)$ . Suppose on the contrary there exist at least one interval  $[r_1, r_2] \subseteq [0, \hat{r}]$  such that  $x_p(r) \leq x_a(r)$ . By the continuity of  $x_a$  and  $x_p$ , it must be that  $x_p(r_1) = x_a(r_1)$ . Note that

$$\frac{\partial\phi(z, s, x)}{\partial x} = \frac{(U_{xs} - U_{ys})(U_y - U_x) - U_s(2U_{xy} - U_{xx} - U_{yy})}{(U_y - U_x)^2} < 0. \quad (23)$$

In combination with (22), it would follow that  $x'_a(r) < x'_p(r)$  almost everywhere on  $[r_1, r_2]$ , which combined with  $x_a(r_1) = x_p(r_1)$  is a contradiction to  $x_p(r) \leq x_a(r)$  on the interval. Thus,  $x_p(r) > x_a(r)$  almost everywhere on  $[0, \hat{r}]$ .

We next rule out that  $x_p(r) = x_a(r)$  at individual points. Since the previous argument excludes intervals where  $x_p(r) \leq x_a(r)$ , equality is only possible at the isolated points where  $Z_p(r) = Z_a(r)$ . But at any such point  $\tilde{r}$  on  $(0, \hat{r}]$ , as  $Z_p(r) > Z_a(r)$  almost everywhere, we have that  $g_p(Z_p(\tilde{r})) \geq g_a(Z_a(\tilde{r}))$  (remember that  $Z'(r) = 1/g(Z(r))$ ). Now, note that  $Z_p(\tilde{r}) = Z_a(\tilde{r}) = \tilde{z}$ . Next, we invoke the level-indexing approach and consider solutions to the game in terms of endowments  $z$ . Let  $S(z) = H^{-1}(G(z))$ . Write solutions to the differential equation (17) as  $x_p(z)$  and  $x_a(z)$  for the respective distributions of endowments. Then if  $x_p(\tilde{r}) = x_a(\tilde{r})$ , it must be that  $x_p(\tilde{z}) = x_a(\tilde{z})$ .

As  $x_p(r) > x_a(r)$  for  $r$  in  $(\tilde{r} - \epsilon, \tilde{r})$  for some  $\epsilon > 0$ , we must have  $x_p(z) > x_a(z)$  for endowments slightly less than  $\tilde{z}$ . Note that it must hold that  $x'_p(\tilde{r}) = x'_a(\tilde{r})$ , and for the case of  $g_p(\tilde{z}) > g_a(\tilde{z})$ , it must be that  $x'_p(\tilde{z}) > x'_a(\tilde{z})$  so that  $x_p(z)$  crosses  $x_a(z)$  from below, which is a contradiction. This leaves us with the possibility that  $x_p(r) = x_a(r)$  in a non-generic case of  $g_p(Z_p(\tilde{r})) = g_a(Z_a(\tilde{r}))$ . ■

**Proof of Proposition 3:** First, as endowments are (weakly) higher at  $r = 0$ , by the boundary condition (7) the privately optimal performance will be higher ex post  $x_{c,p}(0) \geq x_{c,a}(0)$  as will equilibrium performance at  $r = 0$ . Thus,  $U_p(0) \geq U_a(0)$  (i.e. as the poorest individual has no reduction in endowments she will not be worse off). We have that

$$\frac{1}{g_p(Z_p(r))} = \frac{dZ_p(r)}{dr} \geq \frac{dZ_a(r)}{dr} = \frac{1}{g_a(Z(r))} \text{ for all } r \in [0, 1]$$

In other words,  $Z_p(r)$  is (weakly) steeper than  $Z_a(r)$  on  $[0, 1]$ , so that clearly  $Z_p(r) \geq Z_a(r)$  for  $r \in [0, 1]$ .

Suppose that  $U_p(0) > U_a(0)$ , and suppose, in contradiction to the claim we are trying to prove, that  $U_p(r)$  equals  $U_a(r)$  at least once on  $(0, 1)$ . Denote the first such point as  $r_1 \in (0, 1)$ . It is easy to show that, as  $Z_p(0) \geq Z_a(0)$  and  $G_p \geq_d G_a$ , we have  $Z_p(r) > Z_a(r)$  for all  $r \in (0, 1]$ . Thus, by Corollary 1,  $x_p(r) > x_a(r)$  on  $(0, 1]$ , and it must be that  $y_p(r) < y_a(r)$  in the neighborhood of  $r_1$ . Let  $U_{i,y}(r) = U_y(x_i(r), Z_i(r) - x_i(r), S(r))$  for  $i = a, p$ . Then, as  $dU_y = U_{xy}dx + U_{yy}dy$ , and, given our original assumptions on  $U$ , it must be that  $U_{p,y}(r) > U_{a,y}(r)$  in the neighborhood of  $r_1$ . Using the marginal utility condition (8), combined with the fact that, given the dispersion order,  $g(Z_p(r)) \leq g(Z_a(r))$ , it must be that  $U'_p(r) > U'_a(r)$  in the neighborhood of  $r_1$ , so that  $U_p(r)$  can only be steeper than  $U_a(r)$ , and thus can only cross from below. Given  $U_p(0) > U_a(0)$ , we are done.

Suppose, instead, that we have that  $U_p(0) = U_a(0)$ . Then, the above argument rules out that  $U_p$  can cross  $U_a$  from above, so that the claim can only fail if there is an interval  $(0, \tilde{r})$  on which  $U_p(r) \leq U_a(r)$ . Then, there must exist a point  $r_2 \in (0, \tilde{r})$  such that  $U'_p(r_2) \leq U'_a(r_2)$  and  $U_{p,y} \leq U_{a,y}$ . But given (8) and also that  $G_p \geq_d G_a$ , if  $U'_p(r_2) \leq U'_a(r_2)$  then  $U_{p,y}(r_2) \leq U_{a,y}(r_2)$ , which can only happen if  $y_p(r_2) \geq y_a(r_2)$ . But this, combined with the fact that  $x_p(r_2) > x_a(r_2)$  (by Proposition 2) implies that  $U_p(r_2) > U_a(r_2)$ , which is a contradiction. ■

**Proof of Proposition 4:** From Proposition 2, we have  $x_p(r) > x_a(r)$  on  $(0, \hat{r}]$ . But note as here  $Z_p(0) > Z_a(0)$ , the lowest agent has a strictly greater endowment, we have also  $x_p(0) > x_a(0)$  as the cooperative choice, which is the equilibrium choice of the bottom agent by (7), is increasing in endowments. Turning to utility, we can consider two cases. First, suppose that  $x_p(r) \geq x_a(r)$  on  $[\hat{r}, 1]$ . Then, as endowments for individuals with rank  $(\hat{r}, 1]$  are strictly lower ex-post than ex-ante, we have necessarily  $y_p(r) < y_a(r)$  on  $[\hat{r}, 1]$ . Now, as  $x_p(r) \geq x_a(r)$  and  $y_p(r) < y_a(r)$ , then for some  $\tilde{r}$  we can find a pair  $(\tilde{x}, \tilde{y})$  such that  $\tilde{x} + \tilde{y} = x_p + y_p$  (that is,  $(\tilde{x}, \tilde{y})$  are feasible given ex-post endowments) but  $x_{c,p} < \tilde{x} < x_p$  and  $\tilde{y} = y_a$ . But then,  $U(x_p(r), y_p(r), S(r)) < U(\tilde{x}, \tilde{y}, S(r)) < U(x_a(r), y_a(r), S(r))$ , and the result follows.

Suppose now instead that  $x_p(r) < x_a(r)$  for some  $r$  in  $(r_1, r_2)$  with  $r_1 > \hat{r}$ . If  $y_p(r) \leq y_a(r)$  on that interval, it is clear that  $U_p(r) < U_a(r)$  and we are done. Suppose instead that  $y_p(r) > y_a(r)$  on some interval  $(r_3, r_4)$  with  $r_4 \leq r_2$  (as endowments are lower ex post for  $r > \hat{r}$ , it must be that  $r_3 > r_1$ ). We want to rule out the possibility of  $U_p(r) \geq U_a(r)$  somewhere on this interval. Now, it must be the case that  $U_p(r_3) < U_a(r_3)$  as  $x_p(r_3) < x_a(r_3)$  and  $y_p(r_3) = y_a(r_3)$ . We have  $g_p(r) \geq g_a(r)$  everywhere. Furthermore,  $dU_y = U_{xy}dx + U_{yy}dy$ . Given that  $x$  decreases and  $y$  increases ex post on  $(r_3, r_4)$  and our original assumptions on  $U$ , it can be calculated that, given (8), that  $U'_p(r) < U'_a(r)$  on this interval. Combined with  $U_p(r_3) < U_a(r_3)$ , the result follows. ■

**Proof of Proposition 5:** First, given the boundary condition (7), we have  $x(0) = x_c(0)$ . Note that applying property (v) to the definition of  $x_c(r)$  in (6), we have  $\partial x_c / \partial s \leq 0$  so that given  $S_p(0) < S_a(0)$ , it follows that  $x_p(0) \geq x_a(0)$ . Almost everywhere on  $[0, \hat{r})$ , we have both  $S_a(r) > S_p(r)$  and  $S'_p(r) > S'_a(r)$ . Note that

$$\frac{\partial \phi(z, s, x)}{\partial s} = \frac{U_{ss}(U_y - U_x) - U_s(U_{ys} - U_{xs})}{(U_y - U_x)^2} \leq 0. \quad (24)$$

It immediately follows that if  $x_a(r) = x_p(r)$  anywhere on  $[0, \hat{r})$ , then  $x'_a(r) > x'_p(r)$ . So, there can only be one crossing of  $x_a(r)$  and  $x_p(r)$  on that interval and  $x_p(r)$  must cut  $x_a(r)$  from below. Thus, the only way for the claim to be false is if  $x_p(r) \leq x_a(r)$  on some interval  $[0, r_1]$ . But then, as  $\partial \phi(z, s, x) / \partial x < 0$  by (23) and  $\partial \phi(z, s, x) / \partial s \leq 0$  by (24), and as  $S_p(r) < S_a(r)$  and  $S'_p(r) > S'_a(r)$ , it follows that  $x'_p(r) > x'_a(r)$  on  $[0, r_1]$ , which is a contradiction. ■

**Proof of Proposition 6:** Given additively separable utility, we have  $x_p(0) = x_a(0) = x_c(0)$  as with separable utility the cooperative choice does not depend on  $S(0)$ . The differential equation (5) is now

$$x'(r) = \frac{S'(r)}{V_y(x, Z(r) - x) - V_x(x, Z(r) - x)} \quad (25)$$

Given the dispersion order, we have  $S'_p(r) \geq S'_a(r)$  for all  $r$  and the result is easy to establish using the arguments in the proof of the previous proposition. ■

**Proof of Proposition 7:** As with additive separable utility, we have  $x_p(0) = x_a(0)$  irrespective of  $S_a(0)$  or  $S_p(0)$ . The differential equation is now

$$x'(r) = \frac{S'(r)}{S(r)} \frac{V(x, Z(r) - x)}{V_y(x, Z(r) - x) - V_x(x, Z(r) - x)}.$$

Now, by Lemma 1 in Appendix A, by the star order we have  $S'_p(r)/S_p(r) \geq S'_a(r)/S_a(r)$  for all  $r$ . The proof again then follows that of Proposition 5. ■

**Proof of Proposition 8:** Given the lowest reward  $S(0)$  is higher ex post, we have  $U_p(0) > U_a(0)$ . We divide  $[0, 1]$  into two sets. Let  $I_1$  consist of points where  $x_p(r) \geq x_a(r)$  and  $I_2$  consist of points where  $x_p(r) < x_a(r)$ . Considering  $I_2$ , as rewards are

higher and effort lower, clearly  $U_p(r) > U_a(r)$  on  $I_2$ . Turning to  $I_1$ , here  $x_p(r) \geq x_a(r)$  and hence  $y_p(r) \leq y_a(r)$ . Now, as  $U'(r) = U_y S(r)/g(Z(r))$  and  $dU_y = U_{xy}dx + U_{yy}dy$ , we have  $U'_p(r) > U'_a(r)$  almost everywhere on  $I_1$ . The result follows. ■

**Proof of Proposition 9:** We have  $S_a(r) < S_p(r)$  and  $S'_p(r) < S'_a(r)$  on  $[0, \hat{r}]$ . Thus, by reversing Proposition 5, we have  $x_a(r) > x_p(r)$  on  $(0, \hat{r}]$ . Furthermore, given that  $\hat{r}$  is the first point of crossing, we have  $S_a(r) < S_p(r)$  on  $[0, \hat{r}]$ . It is clear that, as performance is strictly lower and rewards are higher under distribution  $H_p(s)$ , it follows that  $U_p(r) > U_a(r)$ . ■

**Proof of Proposition 10:** We have by the envelope theorem  $U'(z) = U_y(x(z), z - x(z), S(z))$ . First, we look at (i). Suppose the claim is false, and there exists at least one interval on  $(\underline{z}, \hat{z}]$  where  $U_p(z) > U_a(z)$ . Let us denote the set of points as  $I_U = \{z \leq \hat{z} : U_p(z) > U_a(z)\}$  (possibly disjoint), and let  $z_1 = \inf I_U \geq \underline{z}$ . We can find a  $z_2 \in I_U$  such that  $U_p(z) > U_a(z)$  for all  $z$  in  $(z_1, z_2]$ . Note that since, by the common boundary condition,  $U_p(\underline{z}) = U_a(\underline{z})$ . As  $G_p(z) \leq G_a(z)$ , then  $S_p(z) \leq S_a(z)$  for all  $z \in I_U$ . As rewards are lower, for  $U_p(z) > U_a(z)$  to be possible, it must be the case that  $x_A(z) < x_B(z)$  for all  $z \in I_U$ . But then as  $U'$  is increasing in  $x(z)$  and strictly increasing in  $S(z)$ , we have  $U'_p(z) \leq U'_a(z)$  on  $I_U$ . This, together with  $U_p(z_1) = U_a(z_1)$ , implies  $U_p(z) \leq U_a(z)$  for all  $z \in (z_1, z_2]$ , which is a contradiction. Part (ii) can be established by an identical argument ■

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