Rapid Prototyping of MIMO Algorithms for OFDM WLAN

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***Abstract*— In the last six years, use of multiple input-multiple output (MIMO) systems in wireless links has been extensively studied. The increase in performance that can be achieved with space-time techniques has spurred efforts to integrate this technology into practical systems. Our aim is to analyze how MIMO technology can be applied to orthogonal frequency division multiplexing (OFDM) wireless local area networks (WLANs), one of the fastest-ever growing technologies in the telecommunications industry. A high level of research has been done from the theoretical point of view, but little analysis has been conducted into practical implementation. In that direction, the so-called rapid prototyping will be the central part of our research because it will facilitate the implementation and testing of MIMO algorithms on the selected hardware platforms (field-programmable gate arrays - FPGAs).**

I. INTRODUCTION

The increasing complexity of communication systems, especially in the last few years, is having an impact on their development time. In the past, the process of implementing a system had three main stages: system model, system prototype and working system. Nowadays, the effort of developing a prototype is similar to that of developing the whole product, so that fewer companies implement a prototype to check its viability. Although professional experience confirms that tendency, we believe that prototyping plays an important role in the development of a communication product, both as a demonstrator and as a testing platform. Therefore, our aim is analyze how rapid prototyping could be optimally applied to MIMO OFDM systems, particularly WLANs. Alpha Data, a company that partially sponsors this research, will provide us with the appropriate prototyping platforms [1].

The Institute of Electrical and Electronics Engineers (IEEE) defined in 1999 the IEEE 802.11a standard for the 5GHz band. It uses OFDM and provides data rates up to 54 Mbps [2] (now also used in the 802.11g standard for the 2.4GHz band). The basic principle of OFDM is to split a high-rate data stream into a number of lower-rate streams which are transmitted simultaneously over a number of subcarriers. Since the symbol duration increases for lower-rate parallel subcarriers, the relative amount of time dispersion caused by multipath delay spread is decreased, making OFDM efficient in wireless propagation scenarios. At the same time, inter symbol interference (ISI) is eliminated almost completely by introducing a guard time in every OFDM symbol. In the guard time, the OFDM symbol is cyclically extended to avoid inter carrier interference (ICI) [3]. Fig. 1 shows the real parts of three of the subcarriers and the frequency arrangement of the subcarriers.

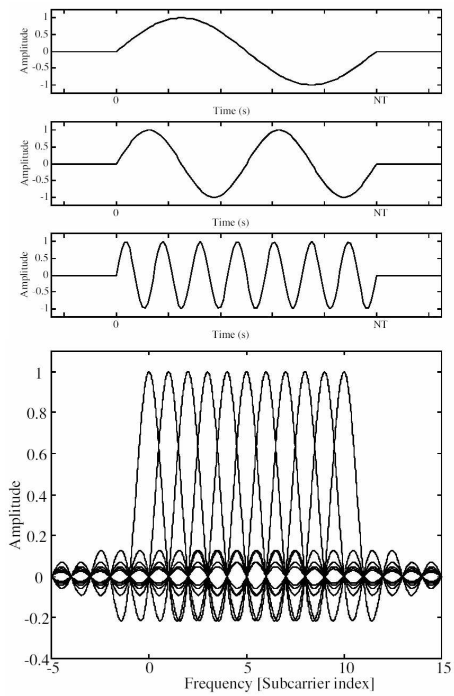

Fig. 1. Subcarriers and frequency spectrum of an OFDM signal

MIMO technology can be used to improve further OFDM systems in terms of spectrum efficiency/capacity, link reliability and coverage. Given an arbitrary wireless communication system, we consider a link for which the transmitting end as well as the receiving end is equipped with multiple antenna elements. The basic idea of MIMO is that the signals on the transmit antennas at one end and the receive antennas at the other end are "combined" in such a way that the quality (bit error ratio - BER) or the capacity (bits per second - bps) will be improved. We will focus on spatial multiplexing systems, that offer a linear (in the number of transmit-receive (TX-RX) antenna pairs or $\min(M, N)$) increase in the capacity for the same bandwidth and with no additional power expenditure. [4]. Fig. 2 shows a schematic representation of an MxN MIMO communication system.

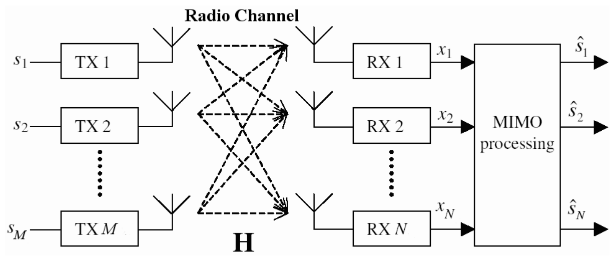

Fig. 2. MIMO communication system with M Tx antennas and N Rx antennas

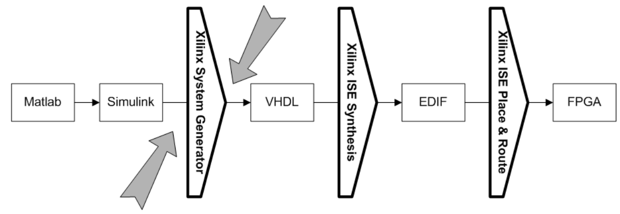

Fig. 3. Hypothetical design flow, the grey arrows show where MIMO concepts can be exploited to improve the final design

This paper is organized as follows: Section II analyzes previous work in the field of rapid prototyping and initial ideas on how it can be used for MIMO systems. Section III describes the OFDM system that is being implemented, discussing channel estimation methods. Section IV studies the different MIMO algorithms that will be considered for prototyping. Finally, Section V concludes the paper, pointing at the future direction of the research.

II. Rapid Prototyping

Developing rapid prototyping systems has been target of both academic research and professional development, yielding several alternatives. Among those alternatives, we can identify two trends: implementing a system using available tools and designing general purpose prototyping tools. They will be described through their most representative examples.

A. Lucent Technologies - TU Wien, Austria

Different communication systems have been implemented using as a prototyping platform a combination of TI digital signal processors (DSPs) and Xilinx FPGAs. They use C as the programming language and use Simulink as the simulation environment. That requires a wrapper to be developed in order to adapt the C code to the S-functions used by Simulink. That C code is refined using specific fixed point libraries and mapped onto the hardware using commercial tools. A wideband code division multiple access (WCDMA) [5] and a modified MIMO system for universal mobile telecommunication system (UMTS) [6] have been implemented using this methodology.

B. Northwestern University, IL, US - AccelChip, CA, US

Northwestern University took a different approach with the Matlab compiler for heterogeneous computing systems (MATCH) project [7]. Instead of implementing a particular communication system on an specific platform, the aim is to obtain a tool able to rapidly prototype communication systems on heterogeneous systems (a combination of embedded processors, FPGAs, DSPs and host processors). Matlab code is used as input and specific Matlab libraries and MATCH directives are used by the compiler to generate code for the different platforms.

From the year 2000, AccelChip has held the exclusive license to develop and distribute products incorporating the MATCH technology [8]. It has evolved to work mainly with FPGA platforms. There are two major limitations of the system. First, Matlab code needs to be written following AccelChip's guidelines, in order to be used by synthesis tools. In addition, the communication system implemented is based on the underlying AccelChip DSP blocks.

C. Proposed Methodology

Our methodology will use Xilinx tools [9] to implement MIMO algorithms for two reasons. Firstly, that will allow us to study in more detail MIMO algorithms and their complexity implications. Secondly, our target platform is a Xilinx FPGA available from Alpha Data. Matlab will be the starting point, using Simulink to have a complete working model of the system. From there, the Xilinx System Generator for DSP will be used to translate the models we want to implement, to very high speed integrated circuit hardware description language (VHDL). The Xilinx Integrated Software Environment (ISE) will be used to synthesize the design and obtain an estimate of the area/gates requirements (important for the study of the MIMO algorithms). That electronic design interchange format (EDIF) information will then be mapped onto the FPGA board.

That methodology follows the Xilinx design flow completely, but our idea is to work on MIMO systems and use their parallelism to our advantage. Thus, we should be able to define an innovative way of rapid prototyping MIMO algorithms. Therefore, somewhere in that design flow, experience from MIMO technology should be applied in order to optimize the results in terms of performance/requirements. We believe that some sort of "intelligent" agent could be attached on the step that goes from the Simulink model to the VHDL/EDIF information. That agent would consider MIMO information to provide an optimized executable. Fig. 3 shows the Xilinx design flow with the grey arrows indicating the positions where MIMO concepts can be applied.

III. OFDM System

Fig. 4 shows the block diagram of a generic OFDM system that has been implemented in Matlab following the IEEE 802.11a standard. The system contains one transmit and one receive antennas where no source or channel coding is used.

The OFDM transmitter first generates equally probable bits for the mapper. The mapper maps the bits according to the selected modulation: binary phase shift keying (BPSK) or quadrature amplitude modulation (4-, 16- and 64-QAM) with symbol energy $E_s = 1$. After the constellation points are obtained, the pilots for frequency correction and channel

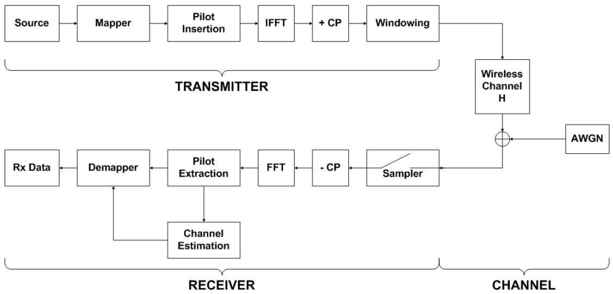

Fig. 4. OFDM system block diagram

estimation are inserted on the specified subcarriers. The inverse fast Fourier transform (IFFT) is performed to obtain the time sequence of the symbol and a cyclic prefix (CP) is added at the beginning of each symbol to avoid ISI. Finally the time sequence is windowed to reduce the out-of-band noise spectrum.

The frames are transmitted through a simple frequency selective channel and contaminated by additive white Gaussian noise (AWGN) with variance $N_0 = \sigma^2$. The channel is 10 samples long (the CP is 16 samples long) and each coefficient is a complex, independent, normally distributed random variable with $\mathrm{E}[|h_n|^2] = 1$. Two power delay profiles have been considered: uniform and exponential power delay profile both with normalized channel response of energy $E_h = 1$. The exponential power delay profile follows (1), where n is the sample number ($\tau_{rms} = 50ns$).

$$\theta[n] = e^{-n} \tag{1}$$

The OFDM receiver, after sampling the signal assuming ideal timing, removes the cyclic prefix and performs an FFT to obtain the constellation points. Before demapping the points, the pilots are extracted to estimate the channel. That estimation information is then used to obtain the sequence of received bits. Fig. 5 shows the bit error ratio (BER) performance of an OFDM system under frequency selective fading with exponential power delay profile, using different modulation schemes where E_b/N_0 is the signal to noise ratio (SNR) per bit. The BER performance degrades when the constellation size increases, while maintaining the same symbol energy.

A. Channel Estimation

The IEEE 802.11a WLAN standard defines two long symbols to be sent during the preamble of the frame in order to facilitate channel estimation. Pilot tones are sent in all the subcarriers that will be used thereafter, requiring block type pilot channel estimation. This type of channel estimation assumes slow fading channels, valid assumption for indoor WLAN systems. Three methods have been studied to perform channel estimation in a 1x1 system.

- Least square (LS) estimation: known symbols $\mathbf{X}_p$ are sent at the transmitter on all the subcarriers of interest. These symbols arrive to the receiver $\mathbf{Y}_p$, where the LS estimation of the channel on those subcarriers is performed following (2).

$$\hat{\mathbf{h}}_{ls} = \frac{\mathbf{Y}_p}{\mathbf{X}_p} \tag{2}$$

The noise and possible interference are not taken into account for the estimation giving poorer performance compared with the other two systems [10].

- Linear minimum mean-square error (MMSE) Estimation: under the assumption that the channel is Gaussian and uncorrelated with the channel noise (of variance σ^2), the LMMSE estimation can be calculated following (3) (after some simplifications that have a negligible effect on the performance [11]).

$$\hat{\mathbf{h}}_{lmmse} = \mathbf{R}_{hh}\left(\mathbf{R}_{hh} + \frac{\beta}{SNR}\mathbf{I}\right)^{-1}\hat{\mathbf{h}}_{ls} \tag{3}$$

$\mathbf{R}_{hh}$ is the autocorrelation of the frequency response of the channel. The factor β is a constant depending on the signal constellation defined by

$$\beta = \mathrm{E}[|X_k|^2]\mathrm{E}[|1/X_k|^2] \tag{4}$$

where X_k is the signal constellation used for the pilot symbols. The factor SNR is defined as $\mathrm{E}[|X_k|^2]/\sigma^2$. The main drawback of this estimator is its high complexity due, mainly, to the matrix inversion.

- Singular value decomposition (SVD) Estimation: in order to reduce the complexity of the LMMSE estimator, a low-rank approximation can be applied using singular value decomposition. The SVD of the channel autocorrelation matrix is shown in (5).

$$\mathbf{R}_{hh} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H \tag{5}$$

$\mathbf{U}$ is a unitary matrix containing singular vectors and $\mathbf{\Lambda}$ is a diagonal matrix containing the singular values

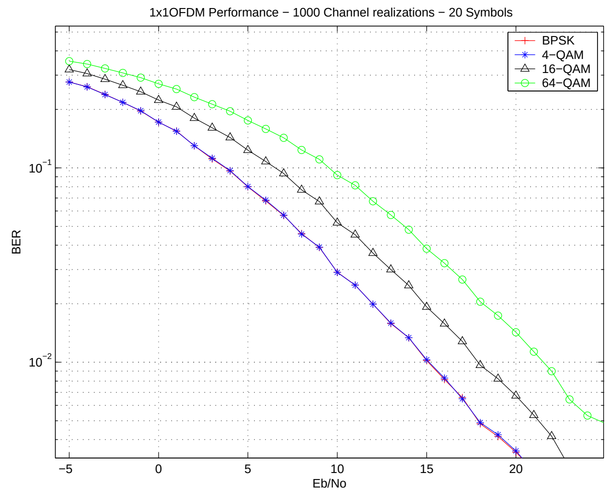

Fig. 5. Bit error ratio performance of an OFDM system under frequency selective fading as a function of signal to noise ratio per bit

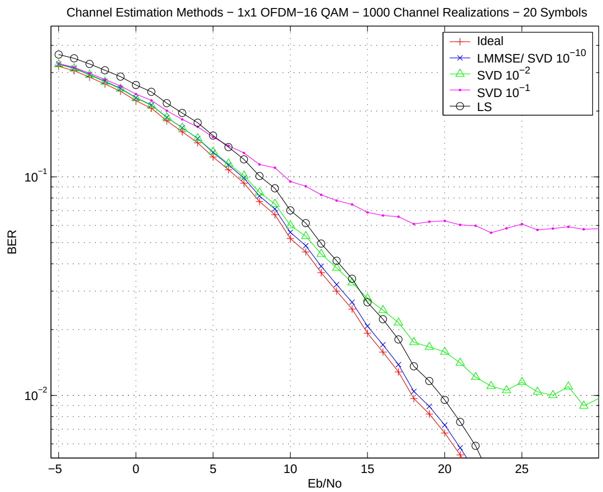

Fig. 6. Bit error ratio performance of OFDM channel estimation methods as a function of signal to noise ratio per bit

$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ on its diagonal [11]. The reduction of complexity from rank-N to rank-p can be achieved using only the p largest eigenvalues instead of all of them, yielding the optimal rank-p estimator shown in (6).

$$\hat{\mathbf{h}}_p = \mathbf{U}\boldsymbol{\Delta}_p\mathbf{U}^H\hat{\mathbf{h}}_{ls} \tag{6}$$

$\boldsymbol{\Delta}_p$ is a diagonal matrix with entries

$$\delta_k = \begin{cases} \frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}}, & k = 1, 2, \cdots, p \\ 0, & k = p+1, \cdots, N. \end{cases} \tag{7}$$

The smaller the number of eigenvalues p is, the lower the computational complexity, but the larger the approximation error becomes.

Fig. 6 shows the performance of the different channel estimation methods applied to the OFDM system using 16-QAM modulation under frequency selective fading with exponential power delay profile. The SVD performance is shown for different number of eigenvalues considered, where only values greater than some percentage of the greatest eigenvalue are used.

It can be seen that the LMMSE/SVD (with eigenvalues greater than 10^{-10} times the largest eigenvalue) channel estimation is the one closest to the ideal channel estimation. On the other hand, the LS has the poorest performance. In addition, the performance of the SVD, when only eigenvalues larger than 10^{-1} and 10^{-2} of the maximum are considered, shows that the approximation error is larger when fewer eigenvalues are considered for the estimation.

IV. MIMO Algorithms

MIMO capabilities and algorithms have been added to the implemented OFDM system. Spatial multiplexing methods (SM) have been used providing a data rate increase. The system includes de-multiplexing and multiplexing units (no coding) with the effect that the information stream is split into multiple parallel systems, which are independently encoded and transmitted simultaneously from the multiple antennas. The following assumptions have been made in order to initially study the different MIMO algorithms:

1) M TX antennas and N RX antennas, $M \leq N$, denoted as MxN system.
2) OFDM transmitters and receivers.
3) The collection of transmitters comprises a vector-valued transmitter in the frequency domain where the same constellation is used for each substream.
4) The power launched by each transmitter is proportional to $1/M$ so that the total radiated power is constant and independent of M.
5) The receivers operate co-channel, each receiving the signals radiated from all M transmit antennas.
6) Rayleigh flat fading is assumed and the matrix channel transfer function is $\mathbf{H}^{NxM}$, where h_{ij} is the complex transfer function from transmitter j to receiver i. The components are complex, independent, normally distributed random variables with $\mathrm{E}[|h_{ij}|^2] = 1$. Note that for the frequency selective fading case, each subcarrier needs to be studied separately, considering flat fading.
7) The system is contaminated by complex AWGN with variance σ^2.

Fig. 7 shows a block diagram of the MIMO-OFDM system. The insertion/extraction of pilots and the channel estimation are not included for the purpose of simplicity.

Assuming symbol-synchronous receiver sampling and ideal timing the received N-vector, using matrix notation, is

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{8}$$

where $\mathbf{s} = [s_1, s_2, ..., s_M]^T$ denotes the vector of transmitted symbols with $\mathrm{E}[{s_i}^2] = 1/M$, $\mathbf{v} = [v_1, v_2, ..., v_N]^T$ the vector of noise samples and $\mathbf{r} = [r_1, r_2, ..., r_N]^T$ the vector of received symbols, all of them in the frequency domain.

We have considered four different algorithms for MIMO detection, providing us with a good range of complexity and performance.

A. ZF Receiver

The zero forcing (ZF) receiver is based on the Moore-Penrose pseudoinverse of the channel and only a way of estimating the channel is required [12].

The definition of pseudoinverse is shown in (9) where $\mathbf{H}^H$ denotes the complex-conjugate transpose of $\mathbf{H}$.

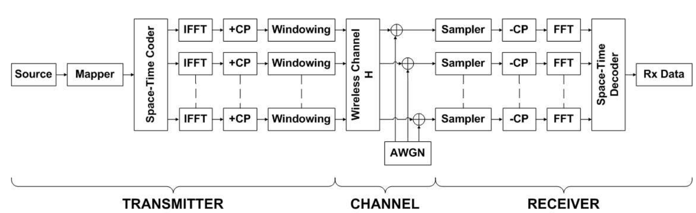

Fig. 7. MIMO-OFDM system block diagram

$$\mathbf{G} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H \tag{9}$$

The symbols before the slicer block are obtained multiplying the received symbols by the pseudoinverse of the channel like in (10) where $\mathbf{n} = \mathbf{G}\mathbf{v}$. In absence of noise, the symbols sent to the slicer are exactly the transmitted symbols.

$$\mathbf{y} = \mathbf{G}\mathbf{r} = \mathbf{s} + \mathbf{n} \tag{10}$$

B. VBLAST-ZF Receiver

The vertical Bell Labs layered space time (VBLAST)-ZF receiver uses the same ZF criterion but the detection is done iteratively. At each symbol time, for each subcarrier, it first detects the strongest layer (depending on the channel matrix) and then cancels the effect of this strongest layer from each of the received signals, considered as interference. The detection continues with the strongest remaining layer, and so on [13].

The optimal detection order is determined by choosing the row of $\mathbf{G}$ with minimum euclidean norm (to maximise the SNR), where $\mathbf{G}$ is defined in (9). The row index is obtained from (11).

$$k = \arg\{\min_i \|\underline{(\mathbf{G})_i}\|^2\} \tag{11}$$

C. VBLAST-MMSE Receiver

The VBLAST minimum mean-square error (MMSE) receiver uses the Wiener equalization of the channel matrix $\mathbf{H}$ instead of the ZF equalization [14]. This receiver balances the mitigation of the interference with noise enhancement, minimizing the total error at the expense of a higher complexity. The equalization matrix $\mathbf{G}$ is calculated using (12).

$$\mathbf{G} = \frac{1}{M}\left(\frac{1}{M}\mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I_M}\right)^{-1}\mathbf{H}^H \tag{12}$$

The optimal detection order is obtained selecting the maximum signal to interference plus noise ratio (SINR) of the transmitted streams still to be decoded on each iteration as expressed on (13).

$$k = \arg\{\max_i (\mathrm{SINR}_i)\} \tag{13}$$

The SINR is calculated on each iteration for each $i \in (1, ..., M)$ transmitted stream, that has not been decoded in previous iterations, using (14). In this equation $\mathbf{g}_i$ is the i-th row of $\mathbf{G}$, $\mathbf{h}_i$ is the i-th column of $\mathbf{H}$, σ^2 is the variance of the noise, E_s is the symbol energy (assumed to be equal for each transmitted antenna) and $\mathbf{i}_{pre}$ is a vector with the indexes of the antennas decoded in the previous iterations.

$$\mathrm{SINR}_i = \frac{(\mathbf{g}_i\mathbf{h}_i)^2 E_s}{\mathbf{g}_i\mathbf{g}_i^H\sigma^2 + \sum_{\forall j \neq i, \mathbf{i}_{pre}}(\mathbf{g}_i\mathbf{h}_j)^2 E_s} \tag{14}$$

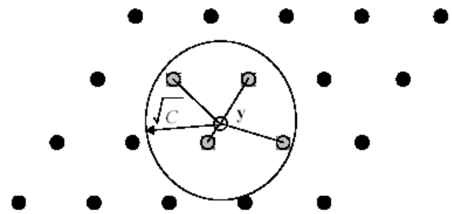

Fig. 8. Schematic of the sphere decoding principle - only the points inside the sphere (grey) are searched

D. Sphere Decoding Receiver

The optimum receiver is the maximum likelihood detector (MLD) but its high complexity makes it unrealizable in practical systems. One method of achieving maximum likelihood (ML) performance with reasonable complexity is to apply the sphere decoder (SD) on the lattice sphere packing representation of a multi-antenna system with no coding [15].

The high complexity of the MLD is due to the search through the entire vector constellation for the most probable transmitted signal vector. It chooses the vector $\mathbf{s}$ that solves (15).

$$\hat{\mathbf{s}} = \arg\{\min_{\mathbf{s}} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2\} \tag{15}$$

On the other hand, the main idea behind SD is to reduce computational complexity by searching over only those lattice points (defined as $\mathbf{H}\mathbf{s}$) that lie within a hypersphere of radius $\sqrt{C}$ around the received signal $\mathbf{r}$, rather than searching over the entire lattice, as shown in Figure 8.

The algorithm follows these steps:

1) The ZF solution for the received signal is chosen as the centre of the hypersphere.
2) According to the radius, the limits of the hypersphere are found in all the dimensions.
3) Following the algorithm described in [16], the points within the previous limits are checked, starting from the lowest values in each dimension (equivalent to start searching from the surface of the hypersphere).
4) When one point is found to be closer to the centre than the surface of the hypersphere, the radius is set to that distance. The algorithm jumps back to step 2.
5) The search stops when no points are found inside the hyperpshere, and the point that set the last radius is chosen as the ML solution.

Fig. 9 gives a possible sequence for finding the ML solution for a simple two dimensional example. Each number (from 1 to 6) indicates a reduction in the radius (6 iterations to find the ML solution).

An important question arises for the SD algorithm, the choice of radius $\sqrt{C}$. If the radius is too large, there are many points to be searched, approaching the ML complexity. On the other hand, if the radius is too small, there might not be any points inside the hypersphere. In practice, the radius is adjusted according to the noise variance of each component ($\sigma^2/2$) so

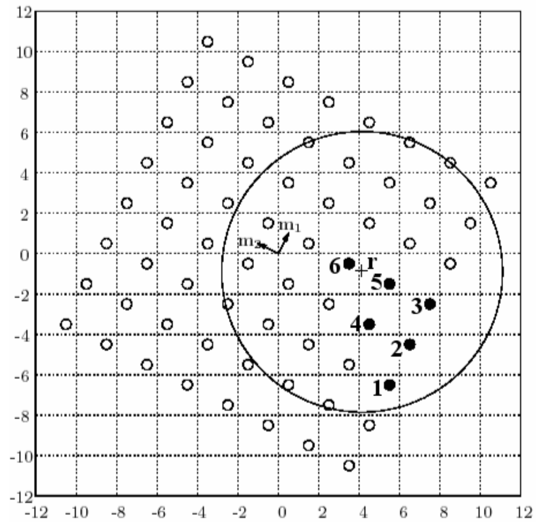

Fig. 9. Operation of the sphere decoding algorithm

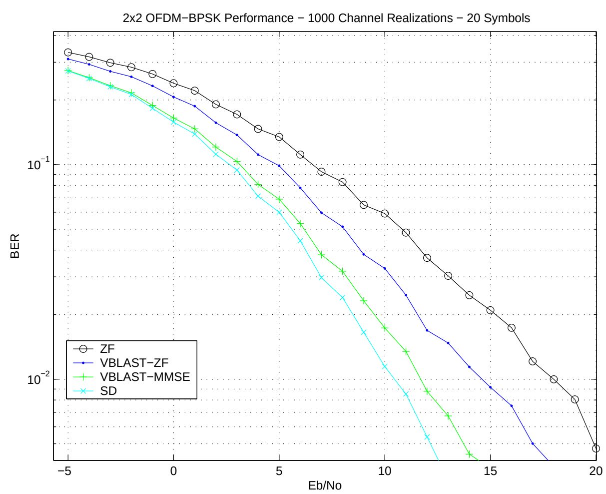

Fig. 10. Bit error ratio performance of MIMO decoding algorithms using OFDM techniques as a function of signal to noise ratio per bit

that the probability of a decoding failure is negligible (e.g. $\sqrt{C} \sim 8\sigma/\sqrt{2}$).

E. Results

Fig. 10 shows the BER performance of the different MIMO algorithms on a 2x2 OFDM system with BPSK modulation. The results have been obtained under flat fading using 1000 channel realizations with 20-symbol frames. The SD algorithm uses an initial radius $\sqrt{C} = 10\sigma/\sqrt{2}$.

The best performance is achieved by the sphere decoder. On the other end, the worst performance is achieved by the ZF receiver where no iterative reception is performed. Between those two curves, we have the VBLAST receivers, with the VBLAST-MMSE performing better than the VBLAST-ZF since it compensates the effect of the noise and the interference. The VBLAST-ZF only takes into account the effect of the channel without considering the noise.

V. Conclusion and Future Work

This paper sets a complete framework for the rapid prototyping of MIMO algorithms for WLAN, from the design methodology to the particular algorithms to be implemented. The selected algorithms provide us with a good range in performance/complexity to test the prototyping methodology that will be defined for MIMO systems. This research will continue in two main directions. Firstly, the channel estimation concepts will be expanded to be used on MIMO systems, identifying the best alternatives in terms of performance/preamble length increase. Secondly, a thorough study of the complexity issues of the MIMO algorithms is required, especially looking at ways of further reducing the complexity of the sphere decoder without compromising its performance.

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